ECE/OPTI 533 Digital Image Processing **Spring Semester, 2003**
Mid-Term Exam 2 Study Guide
April 29, 2003

Exam: Open-Notes, books, no partial credit

This guide is *not* comprehensive; it is intended to be only a practice document for you to help you prepare for the exam

1. Understand how to solve, generate or apply the following:

all the little green "problems" in the notes 15-20

Dan Filiberti's lecture
construct image resolution pyramid
apply Burt generating kernel to create pyramid

apply K-means clustering algorithm to given, small dataset

Given control points, find specified warping function

Given warping function, implement linear or PCC resampling (problem 3 below)

2. Given the following 2-band multispectral image and its covariance matrix C, calculate principal component (eigenimage) 1 and 2

band 1

1	2	2
3	4	4
4	5	6

band 2

2	3	4
4	3	4
6	7	5

C

2.5278	1.7639
1.7639	2.4444

Principal components
From the solution to HW7 for a covariance matrix:

$$C = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$$

eigenvector 1

$$e_{11} = \frac{-b}{\sqrt{(a-\lambda_1)^2 + b^2}} \text{ and } e_{12} = \frac{a-\lambda_1}{\sqrt{(a-\lambda_1)^2 + b^2}}$$

eigenvector 2

$$e_{21} = \frac{-b}{\sqrt{(a-\lambda_2)^2 + b^2}} \text{ and } e_{22} = \frac{a-\lambda_2}{\sqrt{(a-\lambda_2)^2 + b^2}}$$

where the eigenvalues are given by

$$\lambda_k = \frac{a + d \pm \sqrt{(a+d)^2 - 4(ad - b^2)}}{2}$$

Plugging in the numbers from this problem:
eigenvalue 1 = 4.250492842
eigenvalue 2 = 0.721707158
eigenvector e1 (choose positive signs for square root, ala MATLAB)
-0.715413933
-0.698700869
eigenvector e2 (choose negative signs for square root, ala MATLAB)
+0.698700869
-0.715413933

$$W_{pc} = (e1 \;\; e2)^T = \begin{bmatrix} -0.715413933 & -0.698700869 \\ 0.698700869 & -0.715413933 \end{bmatrix}$$

and calculate PC = W_{pc} * DN:

PC1

-2.113	-3.527	-4.226
-4.941	-4.958	-5.657
-7.054	-8.468	-7.786

PC2

-0.732	-0.749	-1.464
-0.766	+0.649	-0.067
-1.498	-1.514	+0.615

3. What is the DN of a resampled pixel at (row,col) = (2.4, 2.1) in the following image using PCC with α = -0.5? Apply the 1-D PCC weighting function as a separable function in 2-D. (15%)

	1	2	3	4
1	203	174	188	163
2	190	156	173	199
3	195	167	197	201
4	191	177	189	221

The offsets of the resample location relative to the original integer pixel grid are 0.4 pixels in rows and 0.1 pixels in columns. Therefore, we will need values of the PCC interpolator at:

across rows (column vector PCCr):
PCCr(-1.4,-0.5) = -0.072
PCCr(-0.4,-0.5) = +0.696
PCCr(+0.6,-0.5) = +0.424
PCCr(+1.6,-0.5) = -0.048

across columns (column vector PCCc):
PCCc(-1.1,-0.5) = -0.0405
PCCc(-0.1,-0.5) = +0.9765
PCCc(+0.9,-0.5) = +0.0685
PCCc(+1.9,-0.5) = -0.0045

Now, you interpolate first across the rows (columns) to get a new row (column), and then interpolate across that new row (column) to get the final result. So, in a compact vector-matrix notation:

$$newrow = PCCr^T \begin{bmatrix} 203 & 174 & 188 & 163 \\ 190 & 156 & 173 & 199 \\ 195 & 167 & 197 & 201 \\ 191 & 177 & 189 & 221 \end{bmatrix} = \begin{bmatrix} 191.1 & 158.4 & 181.3 & 201.4 \end{bmatrix}$$

$$DN_{resample} = PCCc^T \begin{bmatrix} 191.1 & 158.4 & 181.3 & 201.4 \end{bmatrix}^T = 158.5$$

Note, that the order of interpolation does not matter, i.e. you can first interpolate either rows or columns, and then interpolate the new row or column, respectively.