Convective Mass Flows III

- In this lecture, we shall concern ourselves once more with convective mass and heat flows, as we still have not gained a comprehensive understanding of the physics behind such phenomena.
- We shall start by looking once more at the ***capacitive field***.
- We shall then study the ***internal energy*** of matter.
- Finally, we shall look at ***general energy transport phenomena***, which by now include mass flows as an integral aspect of general energy flows.

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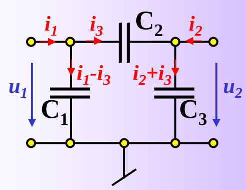
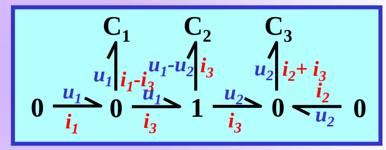
Capacitive Fields III

- Let us briefly consider the following electrical circuit:





$$i_1 - i_3 = C_1 \cdot du_1/dt$$
$$i_2 + i_3 = C_3 \cdot du_2/dt$$
$$i_3 = C_2 \cdot (du_1/dt - du_2/dt)$$

$$\Rightarrow \quad i_1 = (C_1 + C_2) \cdot du_1/dt - C_2 \cdot du_2/dt$$
$$i_2 = - C_2 \cdot du_1/dt + (C_2 + C_3) \cdot du_2/dt$$

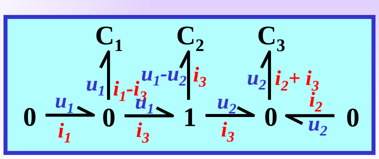
Capacitive Fields IV

$$i_1 = (C_1 + C_2) \cdot du_1/dt - C_2 \cdot du_2/dt$$
$$i_2 = - C_2 \cdot du_1/dt + (C_2 + C_3) \cdot du_2/dt$$

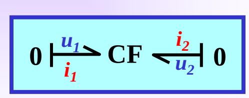
Symmetric capacity matrix

$$\Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (C_1 + C_2) & -C_2 \\ -C_2 & (C_2 + C_3) \end{bmatrix} \cdot \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} du_1/dt \\ du_2/dt \end{bmatrix} = \frac{\begin{bmatrix} (C_2 + C_3) & C_2 \\ C_2 & (C_1 + C_2) \end{bmatrix}}{C_1 C_2 + C_1 C_3 + C_2 C_3} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

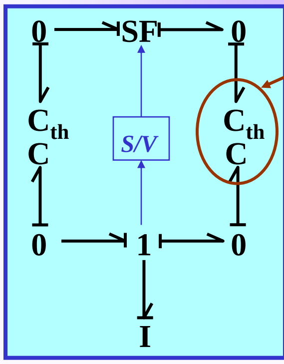


$\Rightarrow$



Volume and Entropy Storage

- Let us consider once more the situation discussed in the previous lecture.



It was no accident that I drew the two capacitors so close to each other. In reality, the two capacitors together form a two-port capacitive field. After all, heat and volume are only two different properties of one and the same material.

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The Internal Energy of Matter I

- As we have already seen, there are three different (though inseparable) storages of matter:
  - Mass
  - Volume
  - Heat
- These three storage elements represent different storage properties of one and the same material.
- Consequently, we are dealing with a ***storage field***.
- This storage field is of a capacitive nature.
- The capacitive field stores the ***internal energy of matter***.

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The Internal Energy of Matter II

- Change of the internal energy in a system, i.e. the total power flow into or out of the capacitive field, can be described as follows :

$$\dot{U} = T \cdot \dot{S} - p \cdot \dot{V} + \sum_{\forall i} \mu_i \cdot \dot{N}_i$$

Flow of internal energy ($\dot{U}$); *Heat flow* ($\dot{S}$); *Volume flow* ($\dot{V}$); *Chemical potential* (μ_i); *Mass flow*; *Molar mass flow* ($\dot{N}_i$)

- This is the ***Gibbs equation***.

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The Internal Energy of Matter III

- The internal energy is proportional to the the total mass n.
- By normalizing with n, all extensive variables can be made intensive.

$$u = \frac{U}{n} \qquad s = \frac{S}{n} \qquad v = \frac{V}{n} \qquad n_i = \frac{N_i}{n}$$

- Therefore:

$$\frac{d}{dt}(n \cdot u) = T \cdot \frac{d}{dt}(n \cdot s) - p \cdot \frac{d}{dt}(n \cdot v) + \sum_{\forall i} \mu_i \cdot \frac{d}{dt}(n \cdot n_i)$$

$$\Rightarrow \boxed{\frac{d}{dt}(n \cdot u) - T \cdot \frac{d}{dt}(n \cdot s) + p \cdot \frac{d}{dt}(n \cdot v) - \sum_{\forall i} \mu_i \cdot \frac{d}{dt}(n \cdot n_i) = 0}$$

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The Internal Energy of Matter IV

$$\frac{d}{dt}(n \cdot u) - T \cdot \frac{d}{dt}(n \cdot s) + p \cdot \frac{d}{dt}(n \cdot v) - \sum_{\forall i} \mu_i \cdot \frac{d}{dt}(n \cdot n_i) = 0$$

$$\Rightarrow \quad n \cdot \left[\frac{du}{dt} - T \cdot \frac{ds}{dt} + p \cdot \frac{dv}{dt} - \sum_{\forall i} \mu_i \cdot \frac{dn_i}{dt}\right] + \frac{dn}{dt} \cdot \left[u - T \cdot s + p \cdot v - \sum_{\forall i} \mu_i \cdot n_i\right] = 0$$

This equation must be valid independently of the amount n, therefore:

Finally, here is an explanation, why it was okay to compute with funny derivatives.

$$\frac{du}{dt} - T \cdot \frac{ds}{dt} + p \cdot \frac{dv}{dt} - \sum_{\forall i} \mu_i \cdot \frac{dn_i}{dt} = 0$$ *Flow of internal energy*

$$u - T \cdot s + p \cdot v - \sum_{\forall i} \mu_i \cdot n_i = 0$$ *Internal energy*

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The Internal Energy of Matter IV

$$U = T \cdot S - p \cdot V + \sum_{\forall i} \mu_i \cdot N_i$$

$$\Rightarrow \quad \dot{U} = T \cdot \dot{S} - p \cdot \dot{V} + \sum_{\forall i} \mu_i \cdot \dot{N}_i + \dot{T} \cdot S - \dot{p} \cdot V + \sum_{\forall i} \dot{\mu}_i \cdot N_i$$

$$= T \cdot \dot{S} - p \cdot \dot{V} + \sum_{\forall i} \mu_i \cdot \dot{N}_i$$

$$\Rightarrow \quad \dot{T} \cdot S - \dot{p} \cdot V + \sum \dot{\mu}_i \cdot N_i = 0$$

- This is the ***Gibbs-Duhem equation***.

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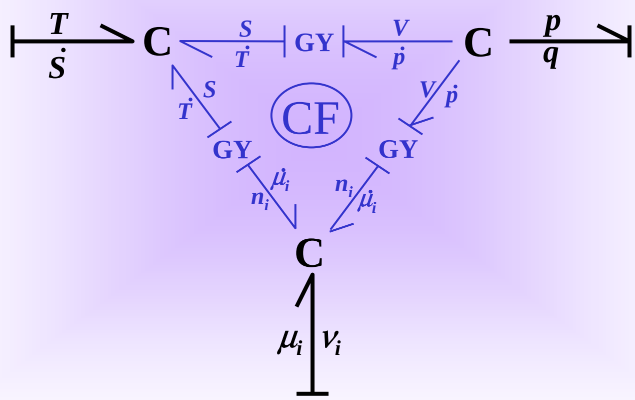
The Capacitive Field of Matter



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Simplifications

- In the case that no chemical reactions take place, it is possible to replace the ***molar mass flows*** by conventional ***mass flows***.
- In this case, the ***chemical potential*** is replaced by the ***Gibbs potential***.

$$\frac{dU}{dt} = T \cdot \dot{S} - p \cdot \dot{V} + g \cdot \dot{M}$$

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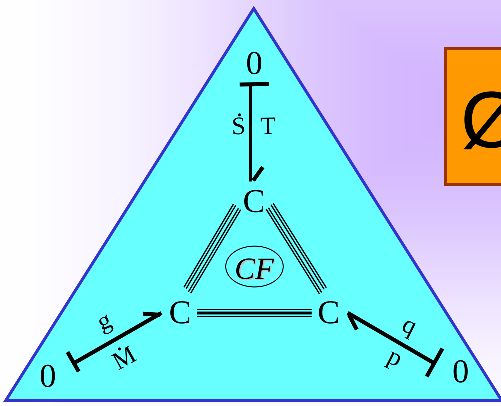
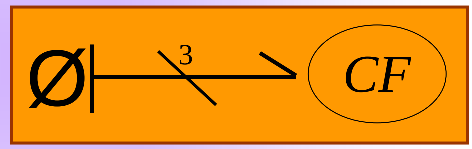
Bus-Bond and Bus-0-Junction

- The three outer legs of the CF-element can be grouped together.





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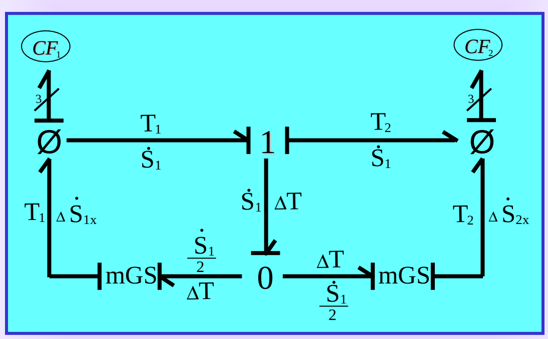
Once Again Heat Conduction



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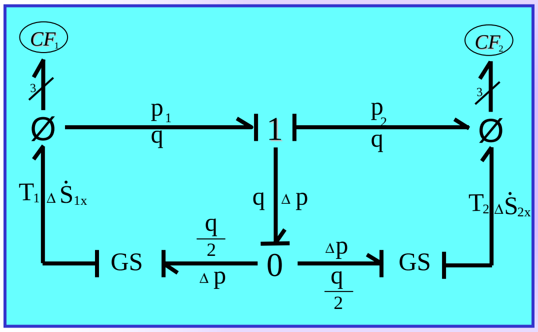
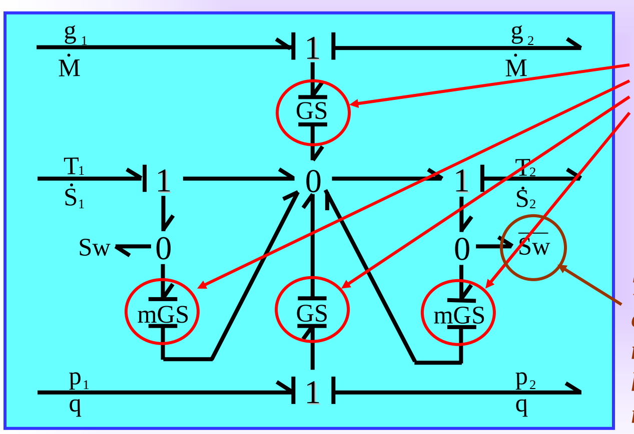
Volume Pressure Exchange



Pressure is being equilibrated just like temperature. It is assumed that the inertia of the mass may be neglected (relatively small masses and/or velocities), and that the equilibration occurs without friction.

The model makes sense if the exchange occurs locally, and if not too large masses get moved in the process.

General Exchange Element I

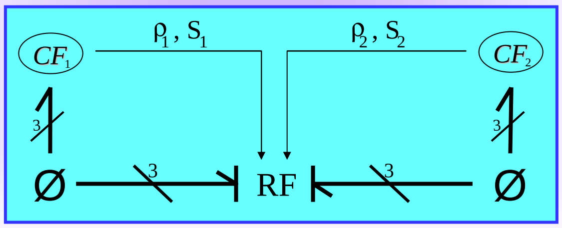


The three flows are coupled through RS-elements.

This is a switching element in bond-graph notation. This element has not yet been introduced.

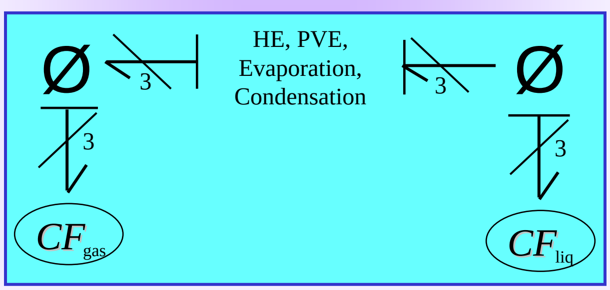
General Exchange Element II

- In the general exchange element, the temperatures, the pressures, and the Gibbs potentials of neighboring media are being equilibrated.
- This process can be interpreted as a ***resistive field***.



Multi-phase Systems

- We may also wish to study phenomena such as ***evaporation*** and ***condensation***.

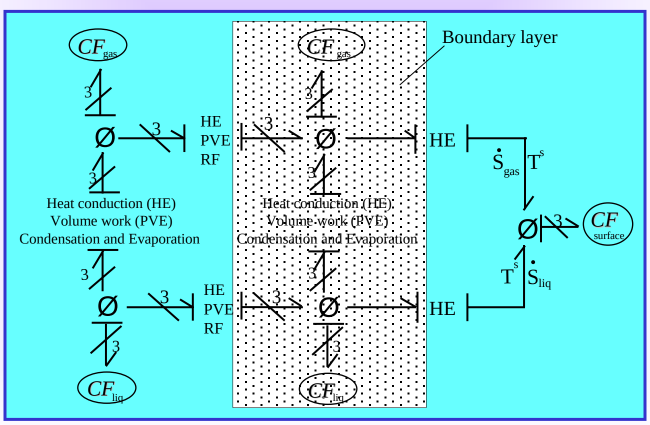


Evaporation (Boiling)

- Mass and energy exchange between capacitive storages of matter (***CF-elements***) representing different ***phases*** is accomplished by means of special resistive fields (***RF-elements***).
- The mass flows are calculated as functions of the pressure and the corresponding saturation pressure.
- The volume flows are computed as the product of the mass flows with the saturation volume at the given temperature.
- The entropy flows are superposed with the enthalpy of evaporation (in the process of evaporation, the thermal domain loses heat → ***latent heat***).

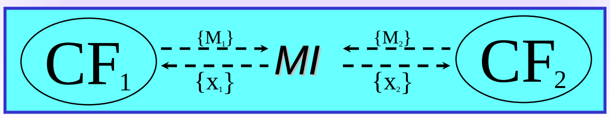
Condensation On Cold Surfaces

- Here, a boundary layer must be introduced.



Thermodynamics of Mixtures

- When fluids (gases or liquids) are being mixed, additional entropy is generated.
- This ***mixing entropy*** must be distributed among the participating component fluids.
- The distribution is a function of the ***partial masses***.
- Usually, neighboring ***CF-elements*** are not supposed to know anything about each other. In the process of mixing, this rule cannot be maintained. The necessary information is being exchanged.



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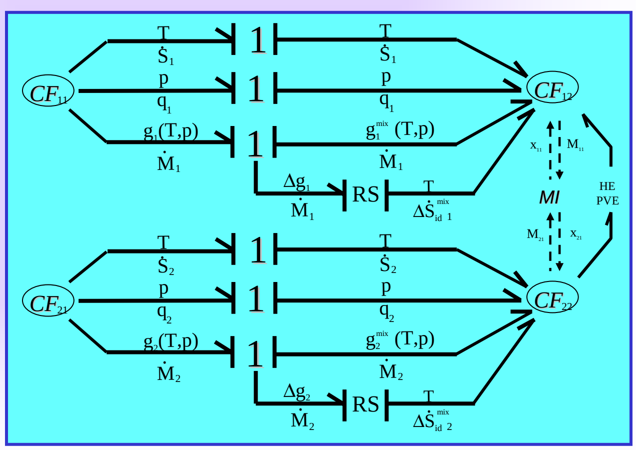
Entropy of Mixing

- The mixing entropy is taken out of the Gibbs potential.

It was assumed here that the fluids to be mixed are at the same temperature and pressure.



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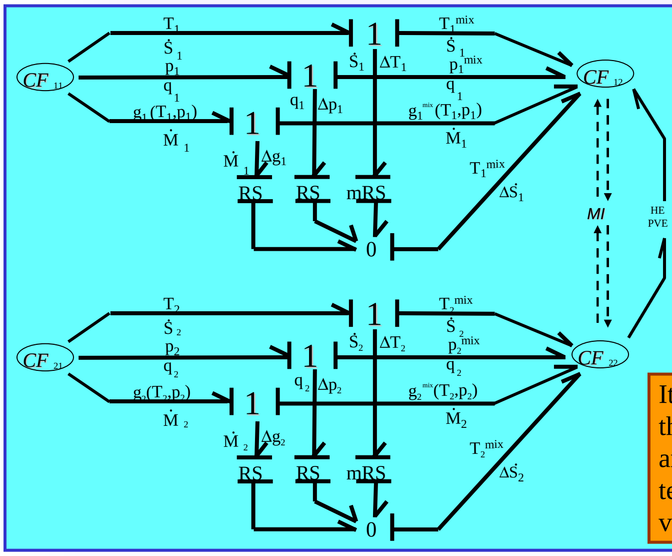

It is also possible that the fluids to be mixed are initially at different temperature or pressure values.

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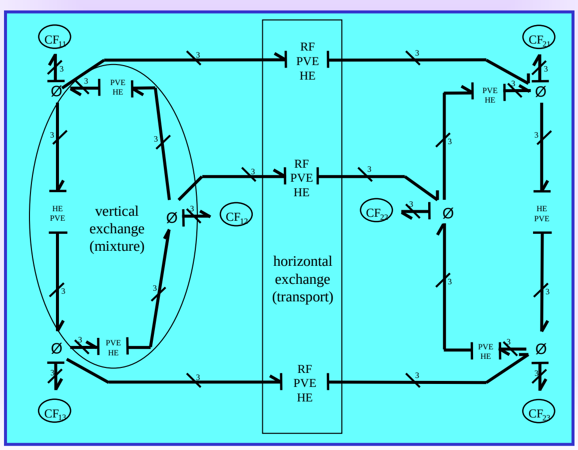
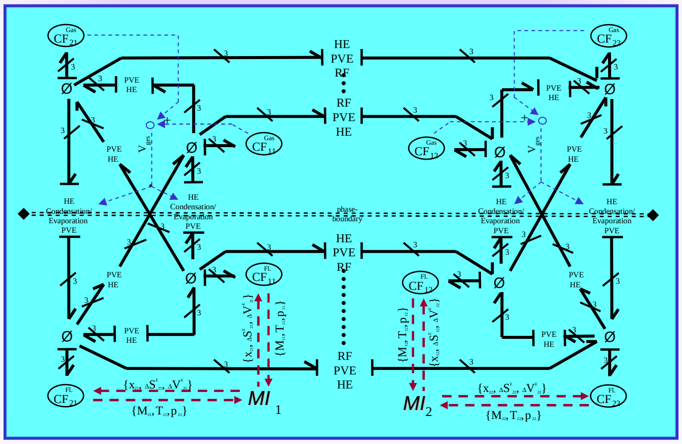
Convection in Multi-element Systems

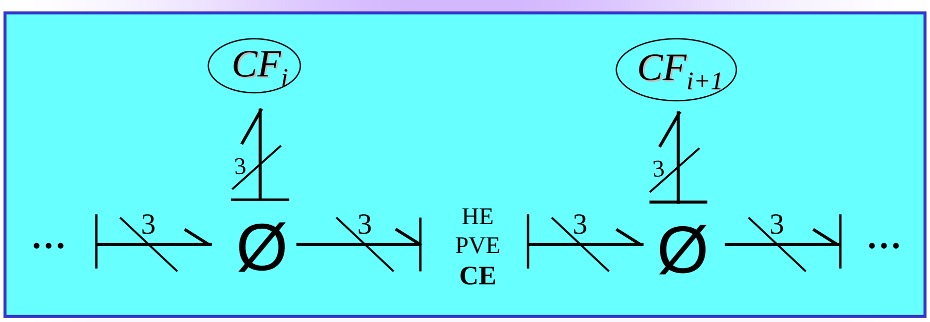


Two-element, Two-phase, Two-compartment Convective System



Concentration Exchange

- It may happen that neighboring compartments are not completely homogeneous. In that case, also the concentrations must be exchanged.



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