

New Matrix Algorithm for Calculating Diagonally Matched Impedance of Packaging Interconnecting Lines

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Abstract

A novel numerical algorithm is developed for computation of diagonally matched impedance matrix of multiple coupled interconnecting lines in high-speed digital circuits. The algorithm is based on the properties of capacitance and inductance matrices of transmission lines and has unconditional monotonic convergence. The mathematical development is based on the theory of M-matrices. The algorithm was applied to various metal lines with different dimensions showing very good accuracy in comparison to other methods.

1. Introduction

Design of fast electronic circuits and systems requires careful analysis of interconnections, which are usually modeled as multiple, coupled transmission lines with discontinuities such as bends, vias, taps and terminations [1] modeled by lumped equivalent circuits. In selecting a final model for the interconnections, there is a trade-off between the accuracy and efficiency of computation. Lossy transmission lines depict the transmission related phenomena such as signal delay, cross-talk, and reflections at terminations and discontinuities with adequate accuracy, but this is obtained at the price of complicated calculations. Much simpler in this respect is approximate modeling with the use of lossless transmission lines. Analysis of signal transmission with such a simplification yields upper bounds for cross-talk and reflections, which is acceptable in the design. The only problem requiring caution is a signal delay which is underestimated. However, package delay is not very significant component in overall system delay and the lossless model of interconnections is acceptable. In a package and system design several quantities are calculated on the basis of inductance and capacitance matrices of the model. In this paper a new technique for calculating diagonally matched impedances denoted here Z_{dm} , which are one of the most important quantities in the design, is presented. In electronic packaging each line is terminated by an active circuit which has no direct connections to other terminating circuits, and ideal matching in the case of coupled lines is not possible. Therefore, in the case of coupled lines packaging engineers practice so called diagonal matching defined below. In miniaturized, high speed circuits, the lines are placed in close proximity and coupling is not negligible. The characteristic admittance matrix of lossless transmission line system, composed of n -signal conductors and a reference conductor is

$$Y = L^{-1}(LC)^{1/2} \quad (1.1)$$

and is known to be invertible [2,3]. The symbols C, L denote $n \times n$ square matrices of per unit length capacitance and inductance, respectively. The matrix of reflection coefficients (for resistive terminations) is defined with the use of the admittance matrix as follows:

$$\mathbf{r} = (Y + X)^{-1}(Y - X) \quad (1.2)$$

where X is the admittance matrix representing resistive terminations. In this equation X, Y and ρ are $n \times n$ real matrices. Matrix X is diagonal and its diagonal elements are determined by the condition that the reflection coefficients on main diagonal are zero. Selection of termination matrix, X , such that the coefficients on the main diagonal of reflection coefficient matrix are zero is called diagonal matching. It is extremely difficult to compute X directly from (1.2). Some approximations were proposed for instance in [4], but their applicability is limited. In [5] an optimization method was introduced for finding impedances, and in [6] an alternative method based on the diagonal form of matrices was applied. The new method introduced in this paper is more efficient than earlier techniques, and does not need any optimization or special canonical forms of matrices.

The purpose of this paper is to introduce a new algorithm for computing matrices Z_{dm} , X and ρ using (1.2). The algorithm is iterative, and it always converges to exact solutions. Section 2 of this paper presents the details of the novel matrix algorithm. Section 3 describes the implementation of the algorithm, and gives a sample of computational results.

2. Matrix Algorithm

In this section a new method for solving equation (1.2) is introduced, which does not require the solution of the non-linear algebraic equations resulting from the element-wise expression of equation (1.2). Notice that the characteristic admittance matrix Y of a transmission line system has the following properties: (i) the diagonal elements are all positive and the off-diagonal elements are non-positive; (ii) the inverse Z of Y is non-negative, that is

$$Y^{-1} = Z \geq 0. \quad (2.1)$$

It is well known that conditions (i) and (ii) mean that Y is an M-matrix as defined in [7]. It is also well known that if the diagonal elements of an M-matrix are increased and the off-diagonal elements remain the same, then the matrix remains an M-matrix and the inverse matrix decreases. This property plays the essential role in our algorithm. Similarly to the procedure discussed in [8], we introduce the mapping $F: R^{n \times n} \rightarrow R^{n \times n}$ such that if $A = (a_{ij})$ is an $n \times n$ real matrix, then $F(A)$ is the diagonal part of A .

Since

$$\begin{aligned} (Y + X)^{-1} (Y - X) &= (Y + X)^{-1} ((Y + X) - 2X) \\ &= I - 2(Y + X)^{-1} X \end{aligned}$$

it follows that

$$F((Y + X)^{-1}) = \text{diag} \left(\frac{1}{2x_1}, \dots, \frac{1}{2x_n} \right),$$

where x_i denotes the i th diagonal element of X , $i = 1, 2, \dots, n$. This equation is equivalent to the n -dimensional fixed point problem

$$X = \frac{1}{2} F^{-1}((Y + X)^{-1}).$$

Consider now the iteration process:

$$X^{(k+1)} = \frac{1}{2} F^{-1}((Y + X^{(k)})^{-1}), X^{(0)} = 0. \quad (2.2)$$

The convergence analysis of this algorithm is given as follows.

Theorem. For all $k \geq 0$, $X^{(k+1)} \geq X^{(k)}$, furthermore the iteration sequence is bounded. Therefore it converges to the smallest non-negative fixed point.

Proof. Since Y is an M-matrix, $X^{(1)} \geq 0 = X^{(0)}$. Assume that for some k , $X^{(k)} \geq X^{(k-1)}$. Then the main property of M-matrices implies that

$$X^{(k+1)} = \frac{1}{2} F^{-1}((Y + X^{(k)})^{-1}) \geq \frac{1}{2} F^{-1}((Y + X^{(k-1)})^{-1}) = X^{(k)}.$$

Hence $X^{(k+1)} \geq X^{(k)}$ for all $k \geq 0$. Let X^* be a non-negative fixed point. Obviously, $0 = X^{(0)} \leq X^*$. Assume now that for some k , $X^{(k)} \leq X^*$. Then

$$X^{(k+1)} = \frac{1}{2} F^{-1}((Y + X^{(k)})^{-1}) \leq \frac{1}{2} F^{-1}((Y + X^*)^{-1}) = X^*,$$

therefore $X^{(k)} \leq X^*$ for all $k \geq 0$.

Hence the iteration sequence is bounded, therefore the theorem of monotone convergence implies that it converges to a diagonal matrix. The continuity of the right hand side of (2.2) in the variable $X^{(k)}$ implies that the limit is a fixed point. The last observation implies that this limit is no greater than any other fixed point, which completes the proof.

Based on the theorem, the following algorithm is suggested. Generate the iteration sequence by equation (2.2) starting with $X^{(0)} = 0$, and stop the iteration, when $\|X^{(k)} - X^{(k-1)}\| < \epsilon$, where ϵ is a given error tolerance.

A computer implementation of this algorithm is discussed in the following section.

3. Implementation and Results

The program computing Z_{dm} calculates L and C matrices first for given strip line systems using Green's function method [9]. Other methods for computing the L, C matrices are available. The L and C matrix computation routine begins with the data structure generation, where the conductor cross section described in the input file of the program are divided into several small intervals. The program can generate up to 1000 intervals for one conductor, and an individual interval is considered as a point in computation. The computation of L and C is performed using Green's functions associated with each case of ground plane configuration on a pair of intervals, each of them from different conductor; and the final results are added using the principle of superposition.

The computation of admittance matrix is based on equation (1.1), which involves the square root of the matrix product LC . A procedure for an efficient direct calculation of characteristic admittance matrix is given in [10]. Then X and ρ are calculated using the

algorithm described above. The iteration starts with the 0 matrix. Then it proceeds with the iterative equation (2.2) which stops when two successive steps yield results with difference smaller than a predefined error tolerance.

The algorithm was coded in C language and applied to compute diagonally matched impedance for various interconnecting structures. In one structure composed of 3 perfect metal conductors formed in a layer of homogeneous dielectric sandwiched between two perfect grounding metal plates (an idealized model of one section of multilayer printed wire board) (Fig. 1) we studied the variation of diagonally matched impedance of center conductor with the normalized width (Fig. 2), thickness, and separation (Fig. 3). In another structure, shown in Fig. 4, where the inductance and capacitance matrices of idealized model of lines are:

$$[C] = \begin{bmatrix} 140.136 & -30.789 & -1.907 & -0.382 & -0.195 \\ -30.789 & 153.673 & -30.514 & -1.858 & -0.382 \\ -1.907 & -30.514 & 153.693 & -30.514 & -1.907 \\ -0.382 & -1.858 & -30.514 & 153.673 & -30.789 \\ -0.195 & -0.382 & -1.907 & -30.789 & 140.136 \end{bmatrix}$$

$$[L] = \begin{bmatrix} 497.841 & 164.707 & 76.964 & 41.775 & 25.539 \\ 164.707 & 490.660 & 162.252 & 76.043 & 41.775 \\ 76.964 & 162.252 & 489.489 & 162.252 & 76.964 \\ 41.775 & 76.043 & 162.252 & 490.660 & 164.707 \\ 25.539 & 41.775 & 76.964 & 164.707 & 497.841 \end{bmatrix}$$

we computed the admittance using the algorithm described in [10]. The diagonally matched impedances for the 5 lines computed using the proposed algorithm were:

$$\begin{aligned} Z_{dm}(1,1) &= 58.33 \text{ Ohm} \\ Z_{dm}(2,2) &= 54.47 \text{ Ohm} \\ Z_{dm}(3,3) &= 54.25 \text{ Ohm} \\ Z_{dm}(4,4) &= 54.47 \text{ Ohm} \\ Z_{dm}(5,5) &= 58.33 \text{ Ohm} \end{aligned}$$

It is worth noting the variation of matched impedance with the position in the structure. This gives suggestion for designing of receivers' input impedance for diagonal match.

We also calculated the diagonally matched impedances for the example structure given in [4] where they are called matched load impedances. The results obtained were practically identical to those reported in the referenced paper.

4. Conclusion

Ever decreasing operating voltage of digital circuits requires an accurate estimation of transmission characteristics and of noises due to tightening noise margins. A novel algorithm used to compute diagonally matched impedance matrices for various strip

structures provides the desired increase in accuracy and efficiency. The algorithm is based on the special properties of capacitance and inductance matrices, and the theory of M-matrices. The monotonic convergence is the fundamental property of the iteration procedure. A computer program developed yields the diagonally matched impedance matrix for any given line geometry. The computation results show an excellent accuracy due to the nature of algorithms. This algorithm is readily expandable for transmission line systems with multi-conductor and multi-layer dielectrics, providing a powerful and useful software tool for designing and analyzing high speed circuit structures.

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Figure 2. A diagonally matched impedance (Z_{dm}) curve of the center line of three parallel line structures with a fixed normalized conductor thickness $\left(\frac{t}{h}\right)$ of 0.05.

Figure 3. Diagonally matched impedance (Z_{dm}) curves of the center line of three parallel transmission line structures with a fixed normalized width $\left(\frac{w}{h}\right)$ of 1.0 (curves a) and with a fixed normalized separation $\left(\frac{s}{h}\right)$ of 1.0 (curves b).