Solutions to Homework Set 1

February 11, 2000

1 Problem 1a

We write $f_1(x)$ as:

$$f_1(x) = rect(\frac{x}{w}) * [\delta(x) + \frac{1}{|2w|}\delta\delta(\frac{x}{2w})] \tag{1}$$

The Fourier Transform of $f_1(x)$ is:

$$F_1(u) = w.sinc(w.u).[1 + 2.\tfrac{2w}{2w}.cos(2.\pi.2w.u)]$$

Therefore,

$$F_1(u) = w.sinc(w.u).[1 + 2.cos(4.\pi.w.u)] \tag{2}$$

The amplitude is shown on Figure 2. The phase $Pha(F_1(u))$ is 0 or π depending on the sign of $Re(F_1(u))$.

2 Problem 1b

The are several ways to write $f_2(x)$. One simple way to do is:

$$f_2(x) = \frac{1}{2} - rect(\frac{x}{w}) * [\frac{1}{|2w|}.comb(\frac{x}{2w})] \tag{3}$$

The Fourier Transform of $f_2(x)$ is:

$$F_2(u) = \frac{1}{2}.\delta u - w.sinc(w.u).comb(2w.u) \tag{4}$$

See Figure 3.

Once again the phase $Pha(F_2(u))$ is 0 or π depending on the sign of $Re(F_2(u))$.

3 Problem 2a

Simple case of lowpass filtering on space domain. The resulting function is the $tri(\frac{x}{b})$ with the sides chopped at a width 'b'(see Figure 1)

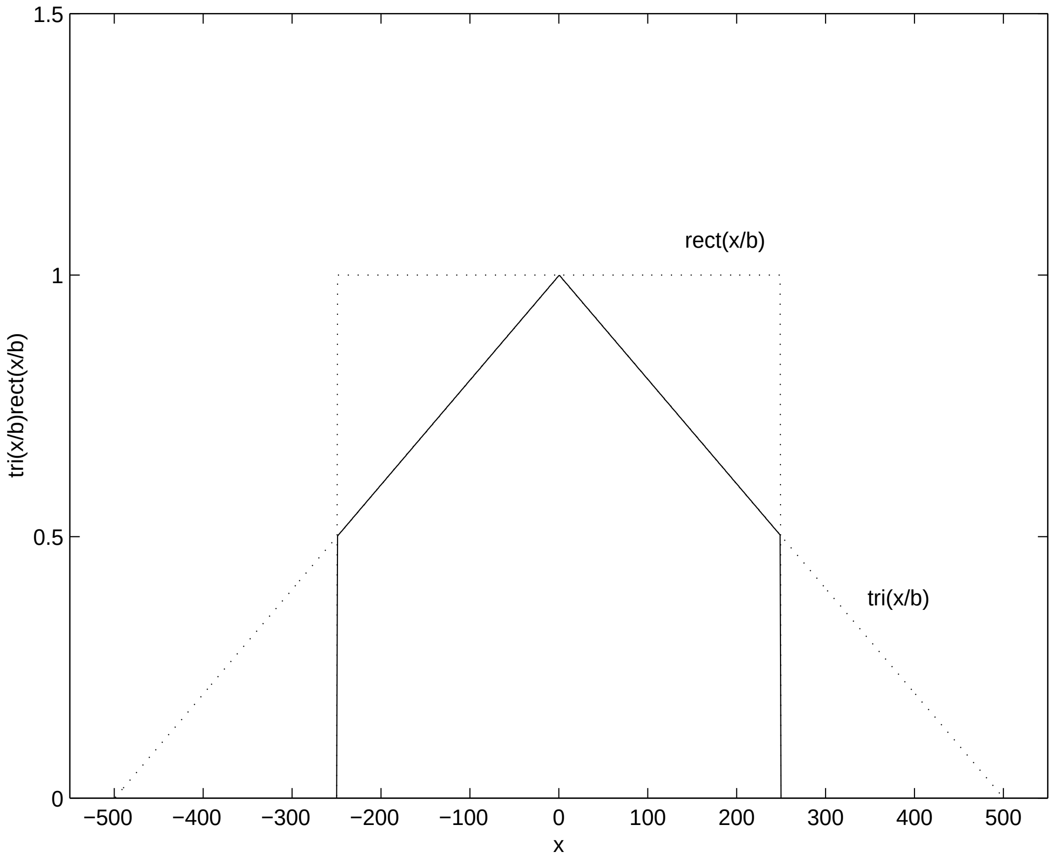

Figure 1: Plots of $rect(\frac{x}{b}).tri(\frac{x}{b})$ for b = 500

4 Problem 2b

$$g_2(x) = cos(\frac{2\pi x}{b}) * \delta\delta(\frac{2x}{b}) \quad (5)$$

We can rewrite the function as:

$$g_2(x) = cos[\frac{2\pi}{b}.(x - \frac{b}{2})] + cos[\frac{2\pi}{b}.(x + \frac{b}{2})] \quad (6)$$

or

$$g(x) = -bcos(\frac{2\pi x}{b}) \quad (7)$$

or, we can find the Fourier Transform of $g_2(x)$ in equation 5:

$$FT[g_2(x)] = \frac{|b|}{2}\delta\delta(b.u).2\frac{|b|}{2}cos(2\pi x\frac{b}{2}.u) = \frac{|b|^2}{2}\delta\delta(b.u).cos(2\pi x\frac{b}{2}.u)$$

$$= -\frac{|b|^2}{2}\delta\delta(b.u) = F_2(u)$$

And find the Inverse Fourier Transform of $F_2(u)$:

$$FT^-1[F_2(u)] = -\frac{|b|^2}{2}\frac{2}{|b|}.cos(\frac{2\pi x}{b}) = -|b|.cos(\frac{2\pi x}{b})$$

See Figure 4.

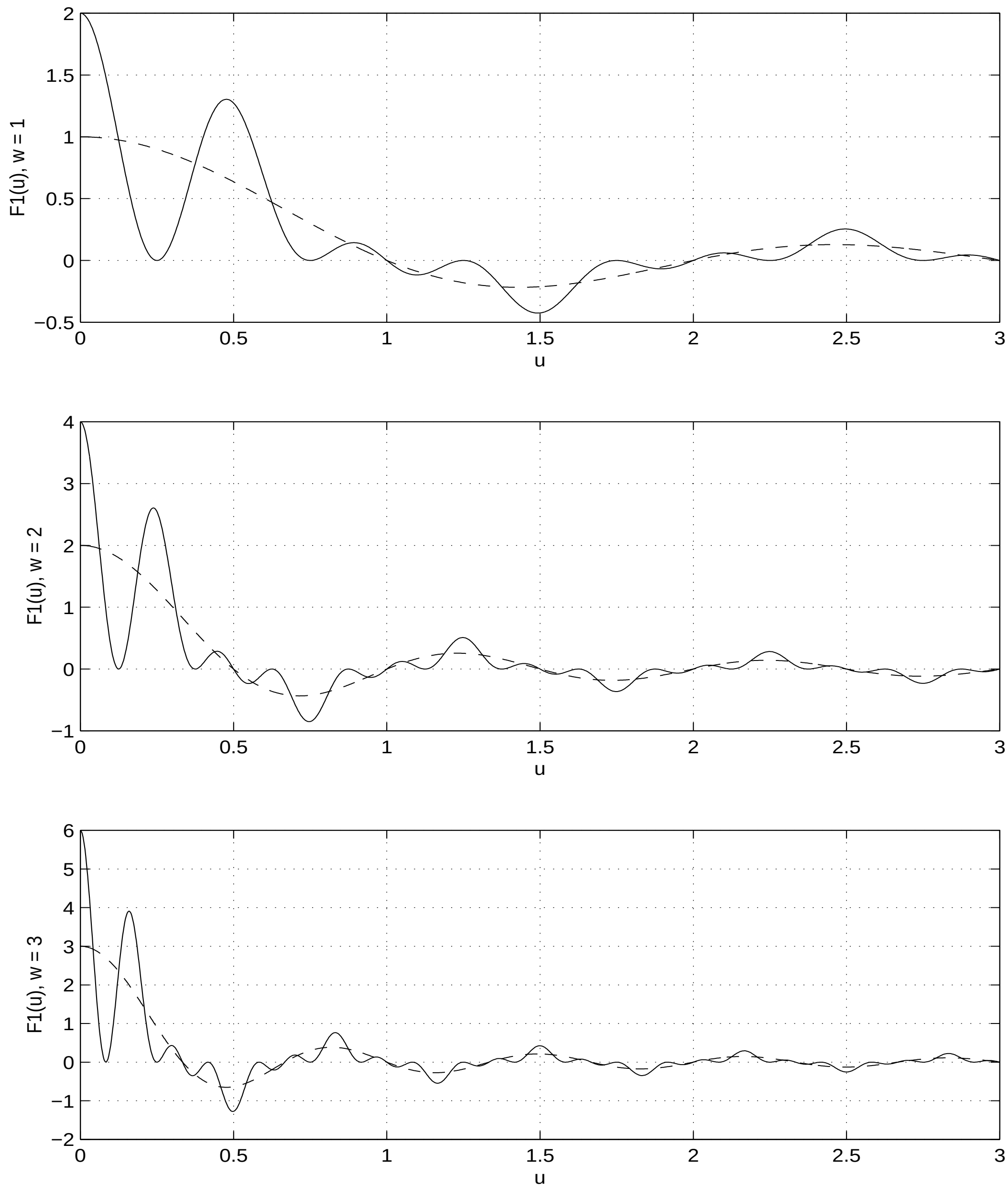

Figure 2: Plots of $F_1(u)$ for 3 different values of w

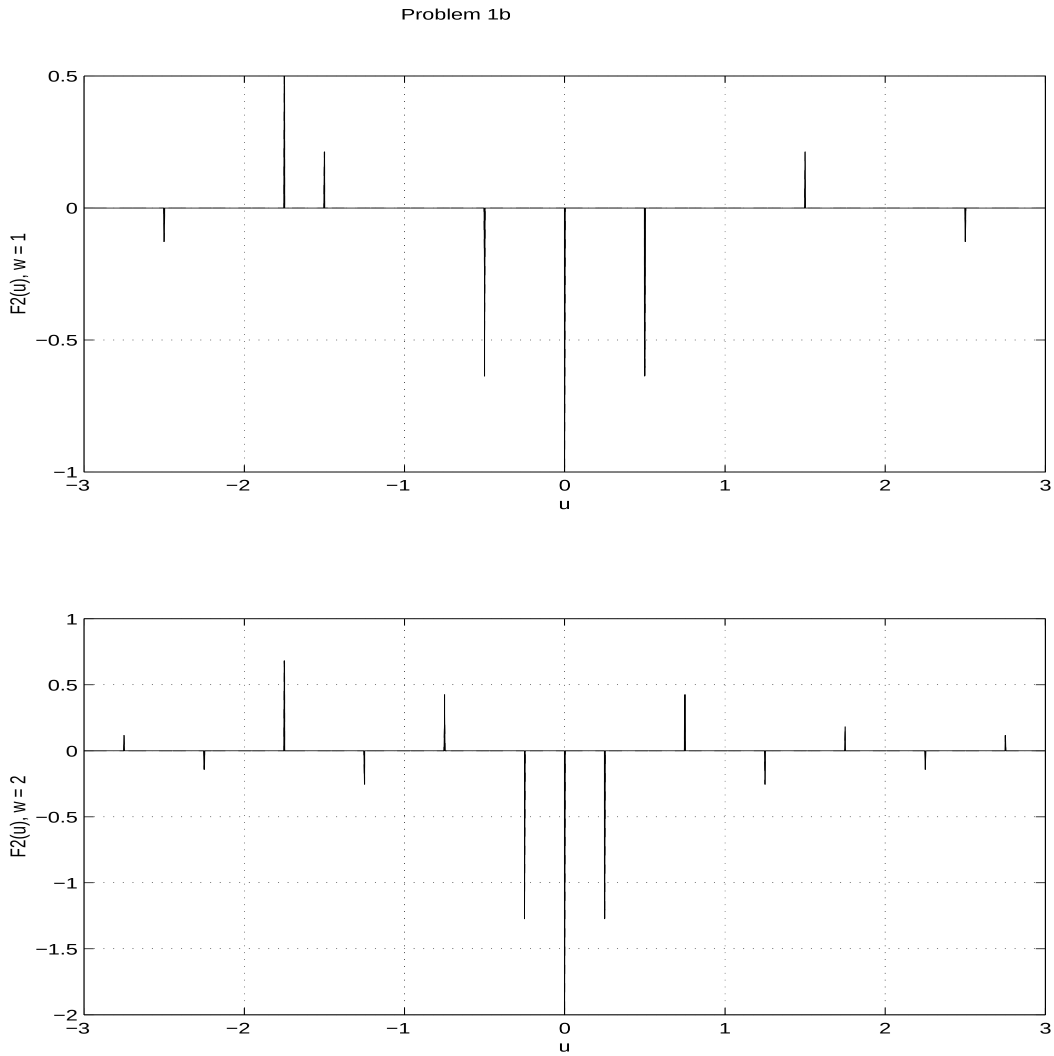

Figure 3: Plots of $F_2(u)$ for 2 different values of w

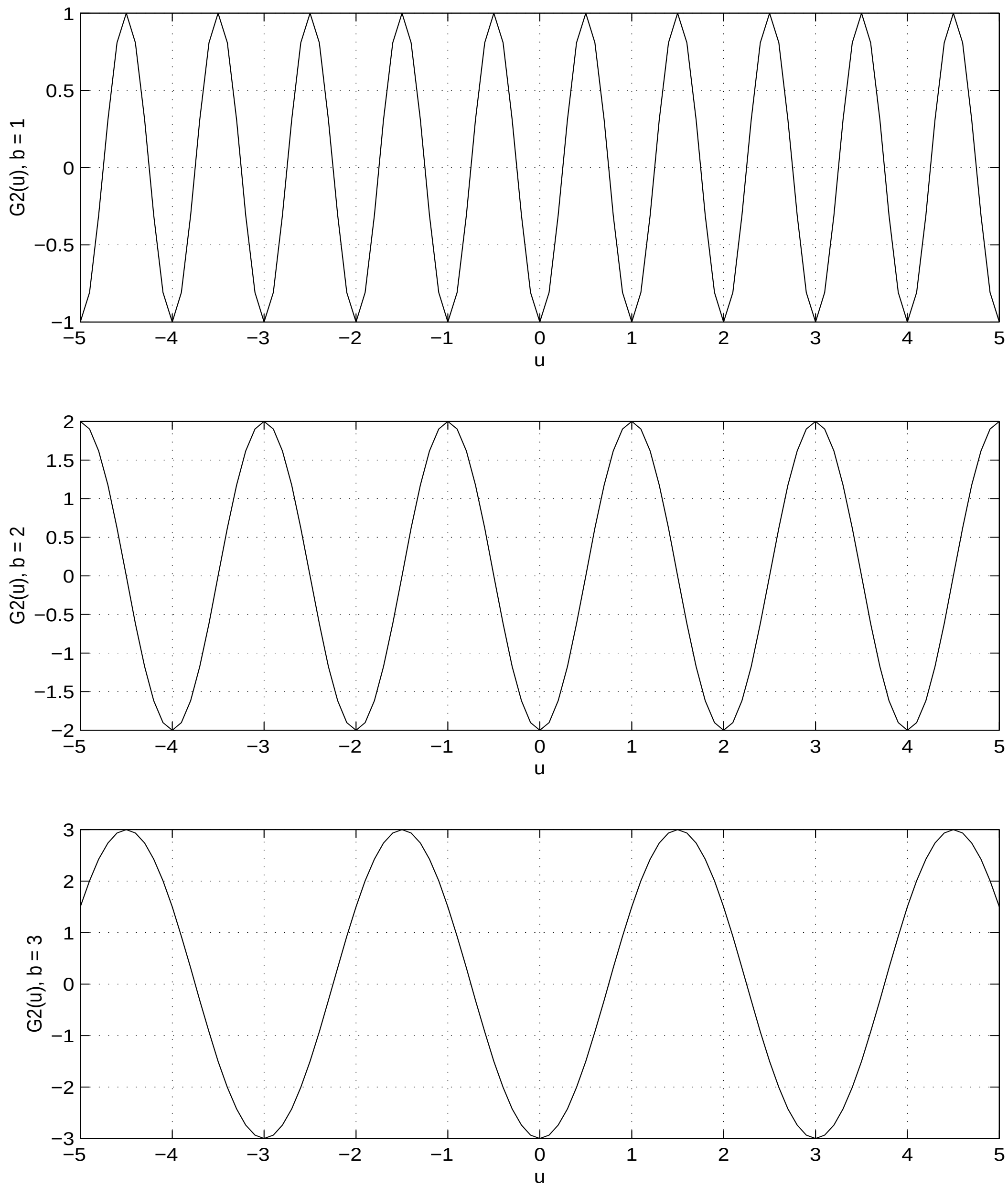

Figure 4: Plots of $G_2(u)$ for 3 different values of b