Solutions to Homework Set 2

February 21, 2000

1 Problem 1

1.1 a) rect(x)

$$F[rect(x)] = \int_{-\infty}^{\infty} rect(x).e^{-2j\pi v\beta} d\beta = \int_{-1/2}^{1/2} (1).e^{-2j\pi v\beta} d\beta = [\frac{e^{-j2\pi v\beta}}{-2j\pi v}]\Big|_{-1/2}^{1/2} =$$

$$= [\frac{e^{-j\pi v} - e^{j\pi v}}{-2j\pi v}] = [\frac{e^{j\pi v} - e^{-j\pi v}}{2j}].(\frac{1}{\pi v}) = \frac{sin(\pi v)}{\pi v} = sinc(v)$$

1.2 b) rect(x)cos($2\pi x$)

$$F[rect(x)cos(2\pi x)] = \int_{-1/2}^{1/2} (1).cos(2\pi\beta) e^{-2j\pi v\beta} d\beta = \frac{1}{2}[\int_{-1/2}^{1/2} (1).(e^{j2\pi\beta} + e^{-j2\pi\beta}) e^{-2j\pi v\beta} d\beta] =$$

$$= \frac{1}{2}[\int_{-1/2}^{1/2} (e^{j2\pi\beta}) e^{-2j\pi v\beta} d\beta + \int_{-1/2}^{1/2} (e^{-j2\pi\beta}) e^{-2j\pi v\beta} d\beta] =$$

$$= \frac{1}{2}[\int_{-1/2}^{1/2} e^{j2\pi\beta(1-v)} d\beta + \int_{-1/2}^{1/2} e^{-j2\pi\beta(1+v)} d\beta] =$$

$$= \frac{1}{2}[(\frac{e^{j2\pi\beta(1-v)}}{2j\pi(1-v)})\Big|_{-1/2}^{1/2} + (\frac{e^{j2\pi\beta(1+v)}}{2j\pi(1+v)})\Big|_{-1/2}^{1/2}] =$$

$$= \frac{1}{2}[(\frac{e^{j\pi\beta(1-v)} - e^{-j\pi\beta(1-v)}}{2j})(\frac{1}{\pi(1-v)}) + (\frac{e^{j\pi\beta(1+v)} - e^{-j\pi\beta(1+v)}}{2j})(\frac{1}{\pi(1+v)})] =$$

$$= \frac{1}{2}[(\frac{sin(\pi(1-v))}{\pi(1-v)}) + (\frac{sin(\pi(1+v))}{\pi(1+v)})] = \frac{1}{2}[(\frac{-sin(\pi(v-1))}{-\pi(v-1)}) + (\frac{sin(\pi(1+v))}{\pi(1+v)})] =$$

$$= \frac{1}{2}[sinc(v-1) + sinc(v+1)]$$

1.3 c) rect(x)cos($2\pi 4x$)

Similar to problem 1b except that the (1-v) and (1+v) terms are replaced by (4-v) and (4+v), respectivelly. The result is:

$$F3(v) = \frac{1}{2}[sinc(v-4) + sinc(v+4)]$$

2 Problem 2

2.1 a) rect(x)cos($2\pi x$)

$$F[rect(x)cos(2\pi x)] = F[rect(x)] * F[cos(2\pi x)] =$$

$$= sinc(v) * \frac{1}{2}(\delta\delta(v)) = sinc(v) * \frac{1}{2}(\delta(v-1) + \delta(v+1)) =$$

$$= \frac{1}{2}[sinc(v-1) + sinc(v+1)]$$

2.2 b) rect(x)cos($2\pi 4x$)

$$F[rect(x)cos(2\pi 4x)] = F[rect(x)] * F[cos(2\pi 4x)] =$$

$$= sinc(v) * \frac{1}{2(4)}(\delta\delta(v)) = sinc(v) * \frac{1}{2}(\delta(v-4) + \delta(v+4)) =$$

$$= \frac{1}{2}[sinc(v-4) + sinc(v+4)]$$

2.3 c) $\frac{1}{2}$[rect(x).comb($\frac{x}{2}$)]

$$F[\frac{1}{2}(rect(x)comb(\frac{x}{2}))] = \frac{1}{2}F[rect(x)] * F[comb(\frac{x}{2})] =$$

$$= \frac{1}{2}sinc(v)2comb(2v) = sinc(v)comb(2v)$$

3 Problem 3

3.1 a) Graphs of Amplitude of Spectra

The graphs of the amplitude of the spectra are shown on Figure 1.

3.2 b) Graphs of Phase of Spectra

The graphs of the phase of spectra are shown on Figure 2.

4 Problem 4

4.1 a) $\frac{1}{2}$[rect(x).comb($\frac{x}{2}$)]

$$f(x) = a_o + \sum_{-\infty}^{+\infty}(a_n cos(nwox) + b_n sin(nwox))$$

The sampling period of the function is: To=2 $\rightarrow wo = \frac{2\pi}{To} = \pi$
The function is even, therefore $b_n = 0$.

$$a_o = \frac{1}{To}\int_{To} f(x)dx = \frac{1}{2}\int_{-1/2}^{1/2} dx = \frac{1}{2}$$

$$a_n = \frac{2}{To}\int_{To} f(x)cos(nwox)dx = \int_{-1/2}^{1/2} cos(n\pi x)dx =$$

$$= [\frac{sin(n\pi x)}{n\pi}] | _{-1/2}^{1/2} = \pm\frac{2}{n\pi}, nodd$$

or 0, for n even.

4.2 b) Compare results from 4a and 2c

The eigenvalues a_n should be equal to the sinc(v) envelope in 2c.

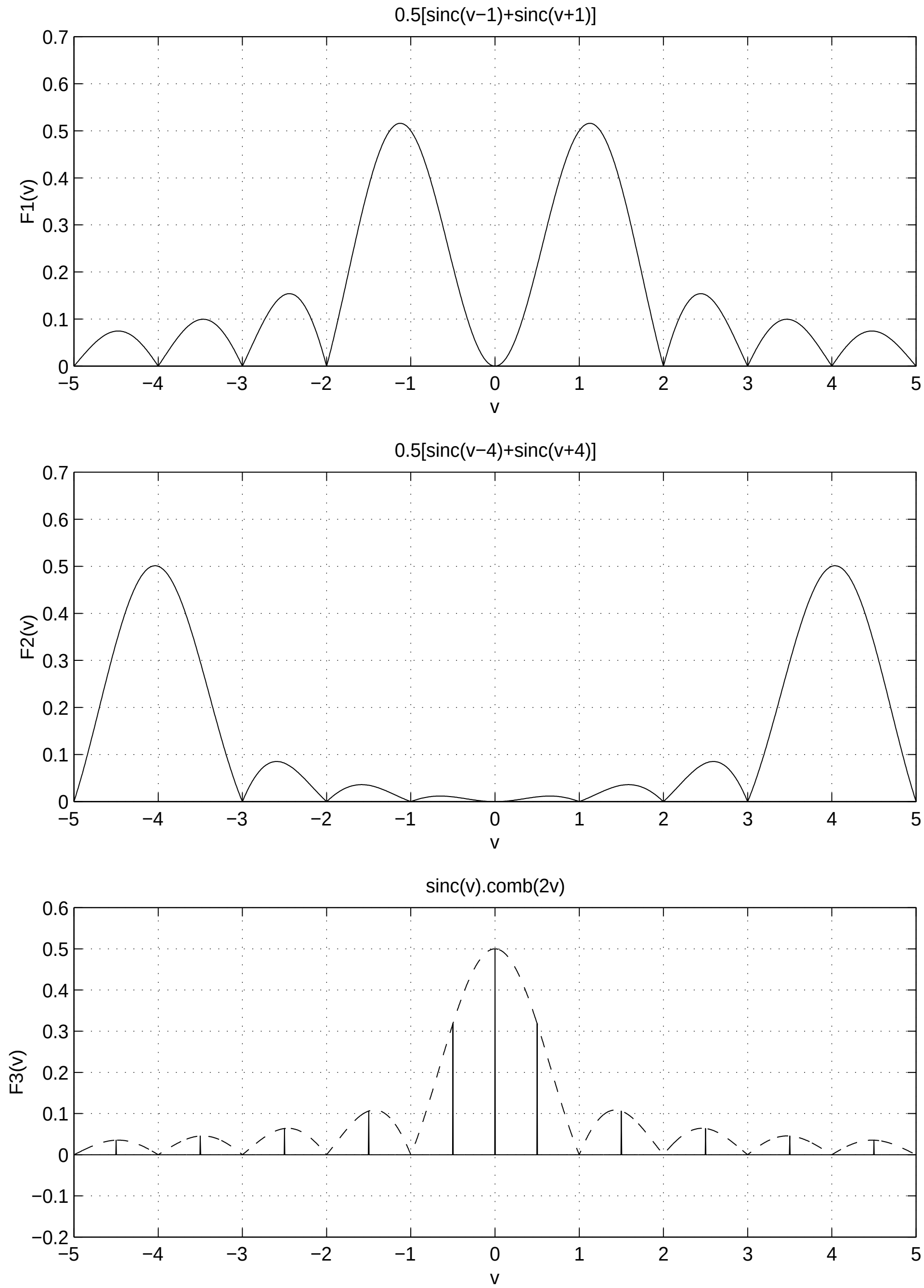

Figure 1: Plots of the Amplitude of the Spectra for problems 2a, 2b, and 2c.

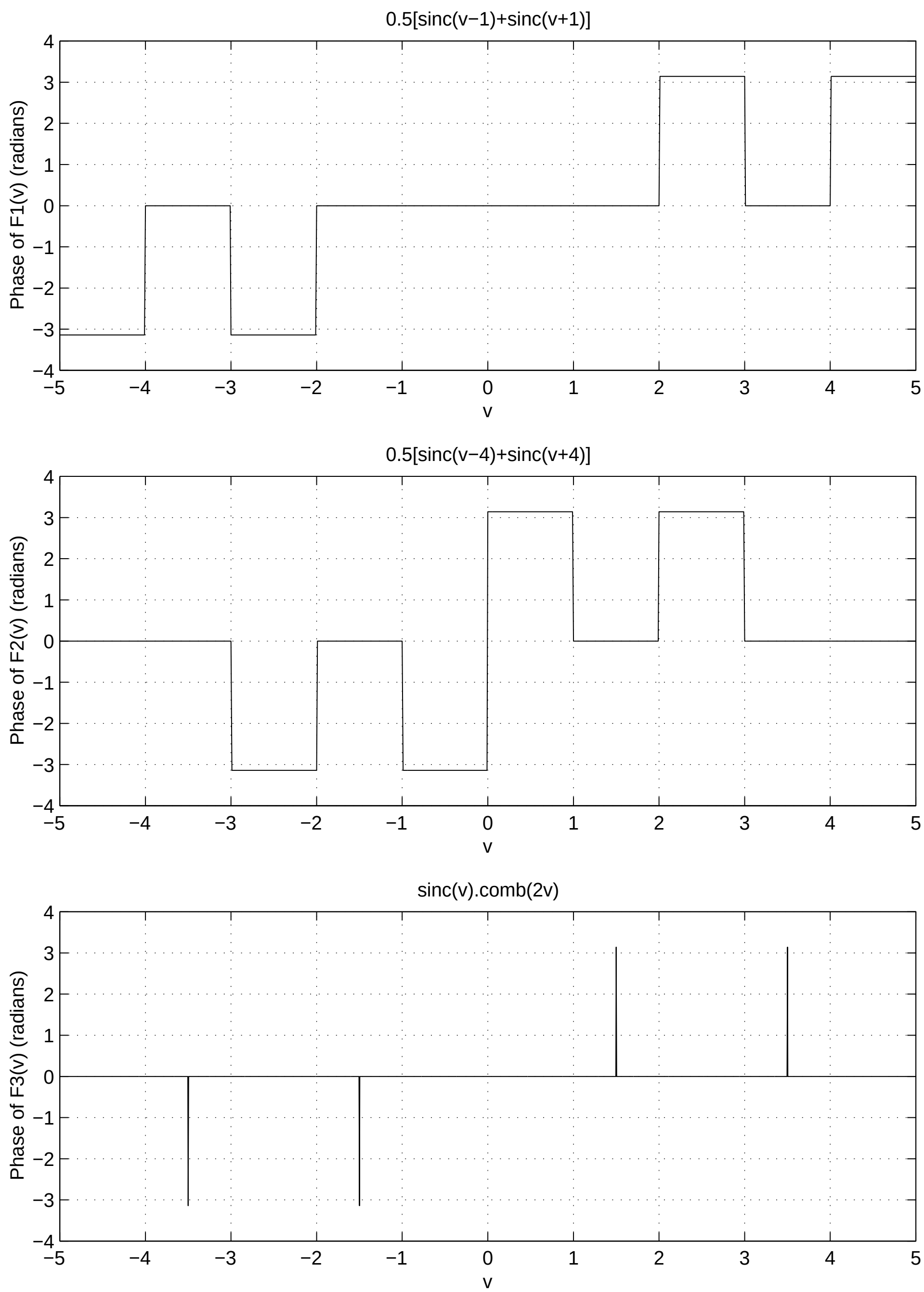

Figure 2: Plots of the Phase of the Spectra for problems 2a, 2b, and 2c.