

ECE 425 – Image Science and Engineering Spring Semester 2000

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Homework #4 Solutions

1. Find the total solar irradiance at the top of the earth's atmosphere.

Assume the sun is a 6000K blackbody. The solar radius is 6.45×10^5 km and the solar-earth distance is 1.5×10^8 km. (15%)

Treat as 2 surfaces (source and receiver), normal to line between them.

Stefan-Boltzmann Law: $M_{tot} = \sigma T^4 = 7.35 \times 10^7$

$$\Phi_e = L_s A_{sun} \Omega_e = \frac{M_{tot}}{\pi} A_s \frac{A_e}{d^2}$$

Lambertian source: $E_e = \Phi_e / A_e = \frac{M_{tot}}{\pi d^2} A_s$
 $= 1359 \text{ (W-m}^{-2}\text{)}$

2. Using the table for radiometric->photometric conversion in the notes, and the trapezoidal rule for integration, show that the total radiant exitance for a 6000K blackbody over the range 400-700nm is $2.761 \times 10^7 \text{ W-m}^{-2}$. (15%)

Sum values of M_λ in the table, weighting the two end points (@400 and 700nm) by 0.5, and multiply by $0.01 \mu\text{m}$ (step size $\Delta\lambda$ converted from nm to μm):

$$M_{tot} = 2.761 \times 10^7 \text{ (W-m}^{-2}\text{)}$$

3. A group of UA engineering students are planning to go to the South Sea island of Bingo-Bingo during Spring Break. Since there is no electricity on Bingo-Bingo, they will design a solar cell-powered generator that they can take with them and set up on the island.

The students estimate that they will need at least 1200KWH to have a good time during their 4 night stay. Now, Bingo-Bingo is on the equator, so it is a good assumption that the Sun will be directly overhead at noon during Spring Break. It can also be assumed that there are 12 hours of nighttime, and that no electricity will be used during the daytime, when the students are asleep.

Since the students have not yet learned atmospheric physics, they choose to ignore atmospheric effects in their design calculation. They also know the following facts:

- Bingo-Bingo is 1km x 1km.
- The efficiency of available solar panels is 0.1 in converting optical W to electrical W at all wavelengths in the range of sensitivity 400 – 1000nm.
- A solar tracking system is expensive and too heavy to transport, so they will install the panels fixed in a horizontal position.
- All power generated during the day will be stored in batteries and used that night. These batteries are already on the island as the result of an ill-fated government project.

How big do the solar panels have to be to allow the students the good time they deserve? Do you think they will have a good time? – why or why not? (70%)

Total solar exitance in 400-1000nm range (T=5900K):

$$M_{tot} = \int_{0.4}^{1.0} M_{\lambda BB} = 4.16 \times 10^7 \text{ (W-m}^{-2}\text{)}$$

Total irradiance at earth:

$$E_0 = \frac{\Phi}{A_e} = \frac{L_s A_s A_e}{\pi d^2 A_e} = \frac{L_s A_s}{\pi d^2} = 7.69 \times 10^2 \text{ (W-m}^{-2}\text{)}$$

Required power /day (batteries charged/discharged each day): $P_{req} = 1200\text{KWH}/4 = 300\text{KWH}$

Integrated power density received per m² over 12-hour daylight period:

$$E(t) = E_0 \cos[\theta(t)] = E_0 \cos\left[\frac{\pi}{2}(1 - t/6)\right]$$

$$PD_{tot} = \int_0^{12} E_0 \cos\left[\frac{\pi}{2}(1 - t/6)\right] dt = 5.87 \text{ (KWH-m}^{-2}\text{)}$$

Power density out of solar panels:

$$PD_{out} = \text{efficiency} \times PD_{tot} = 0.587 \text{ (KWH-m}^{-2}\text{)}$$

Total solar panel area required:

$$A_{req} = P_{req}/PD_{out} = \frac{300}{0.587} = 511 \text{ m}^2$$

which is much less than the $1 \times 10^6 \text{ m}^2$ area of Bingo-Bingo. Therefore, the students will have a good time!