ECE304 Homework #1 Solution

Problem 4.6

a).

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-11}}{15 \times 10^{-9}} = 2.3 \times 10^{-3}\,\mathrm{F/m^2} = 2.3\,\mathrm{fF/\mu m^2}$$

$$K'_n = \mu_n C_{ox} = 550 \times 10^{-4} \times 2.3 \times 10^{-3} = 126.5\,\mu\mathrm{A/V^2}$$

b).

$$i_D = \frac{1}{2}k'_n \frac{W}{L}\left(V_{GS} - V_t\right)^2 \Rightarrow 100 = \frac{1}{2} \times 126.5 \times \frac{16}{0.8}\left(V_{GS} - 0.7\right)^2$$

$$V_{GS} - 0.7 = 0.28 \Rightarrow V_{OV} = 0.28\,\mathrm{V},\ V_{GS} = 0.98\,\mathrm{V}$$

$$V_{DSmin} = V_{GS} - V_t = 0.28\,\mathrm{V}$$

c).

For small V_{DS} (triode region):

$$i_D = k'_n \frac{W}{L} V_{OV} \cdot V_{DS}$$

$$r_{DS} = \frac{V_{DS}}{i_D} = \frac{1}{k'_n \frac{W}{L} V_{OV}} = \frac{1}{126.5 \times 10^{-6} \times \frac{16}{0.8} V_{OV}} = 1000$$

$$\Rightarrow V_{OV} = 0.4\,\mathrm{V}$$

$$\Rightarrow V_{GS} = V_{OV} + V_t = 0.4 + 0.7 = 1.1\,\mathrm{V}$$

Problem 4.7

$$k'_n = \mu_n C_{ox} = \mu_n \frac{\varepsilon_{ox}}{t_{ox}} = 650^{-4} \times \frac{3.45 \times 10^{-11}}{20^{-9}} = 112.1\,\mu\mathrm{A/V^2}$$

a). $V_{DS} < V_{GS} - V_t$: triode region

$$i_D = k'_n \frac{W}{L}\left[\left(V_{GS} - V_t\right)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$i_D = 112.1 \times 10^{-6} \times 10 \times \left[(5 - 0.8) \times 1 - \frac{1}{2} \times 1^2\right] = 4.15\,\mathrm{mA}$$

b). $V_{DS} = V_{GS} - V_t$: edge of saturation

$$i_D = \frac{1}{2}k'_n \frac{W}{L}\left(V_{GS} - V_t\right)^2$$

$$i_D = \frac{1}{2} \times 112.1 \times 10^{-6} \times 10 \times (2 - 0.8)^2 = 0.8\,\mathrm{mA}$$

c). $V_{DS} < V_{GS} - V_t$: triode region

$$i_D = k'_n \frac{W}{L}\left[(V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2\right]$$

$$i_D = 112.1 \times 10^{-6} \times 10 \times \left[(5 - 0.8) \times 0.2 - \frac{1}{2} \times 0.2^2\right] = 0.92\,\text{mA}$$

d). $V_{DS} > V_{GS} - V_t$: saturation region

$$i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$i_D = \frac{1}{2} \times 112.1 \times 10^{-6} \times 10 \times (5 - 0.8)^2 = 9.9\,\text{mA}$$

Problem 4.31:

a). Note that $[(\partial i_D / i_D)/\partial T] = [(\partial i_D/\partial T)/i_D]$

$$i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$\frac{\partial i_D}{\partial T} = \frac{1}{2}\frac{\partial k'_n}{\partial T}\frac{W}{L}(V_{GS} - V_t)^2 + \frac{1}{2} k'_n \frac{W}{L}\left[2(V_{GS} - V_t)\left(-\frac{\partial V_t}{\partial T}\right)\right]$$

$$\frac{\partial i_D}{\partial T} = \frac{1}{2}\frac{\partial k'_n}{\partial T}\frac{W}{L}(V_{GS} - V_t)^2 - k'_n \frac{W}{L}(V_{GS} - V_t)\frac{\partial V_t}{\partial T}$$

$$[(\partial i_D / i_D)/\partial T] = [(\partial i_D/\partial T)/i_D] = \frac{\partial k'_n / k'_n}{\partial T} - 2\frac{\partial V_t}{\partial T}\frac{1}{(V_{GS} - V_t)}$$

b). We have

$$\frac{\partial V_t}{\partial T} = -0.002\,\text{V}/^\circ\text{C}, \quad \frac{(\partial i_D / i_D)}{\partial T} = -0.2\%/^\circ\text{C} = -0.002/^\circ\text{C}$$

$$\Rightarrow -0.002 = \frac{(\partial k'_n / k'_n)}{\partial T} - 2 \times (-0.002) \times \frac{1}{5 - 1}$$

$$\Rightarrow \frac{(\partial k'_n / k'_n)}{\partial T} = -0.003/^\circ\text{C} = -0.3\%/^\circ\text{C}$$

Problem 4.32

Case	Transistor	V_S	V_G	V_D	I_D	Type	Mode	$\mu C_{ox} W/L$	V_t
a	1	0	2	5	100	N	Saturation	200	1
	1	0	3	5	400	N	Saturation	200	1
b	2	5	3	-4.5	50	P	Saturation	400	-1.5
	2	5	2	-0.5	450	P	Saturation	400	-1.5
c	3	5	3	4	200	P	Saturation	400	-1
	3	5	2	0	800	P	Saturation	400	-1
d	4	-2	0	0	72	N	Saturation	100	0.8
	4	-4	0	-3	270	N	Triode	100	0.8

Case a).
transistor 1: $V_{GS} = 2\,\text{V}$, $V_{DS} = 5\,\text{V}$, $I_D = 100\,\mu\text{A}$. This is an NMOS in saturation region.

$$I_{D1} = 100 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_t\right)^2$$

When $V_{GS} = 3\,\text{V}$, $I_D = 400\,\mu\text{A}$. Similarly we have:

$$I_{D2} = 400 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - V_t\right)^2$$

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{100}{400} = \frac{\left(2 - V_t\right)^2}{\left(3 - V_t\right)^2}$$

$$\Rightarrow V_t = 1\,\text{V} \Rightarrow \mu_n C_{ox}\frac{W}{L} = 200\,\mu\text{A/V}^2$$

Case b).
transistor 2: $V_{GS} = 3 - 5 = -2\,\text{V}$, $V_{DS} = -9.5\,\text{V}$. This is a PMOS in saturation region.

$$I_{D1} = 50 = \frac{1}{2}\mu_p\frac{W}{L}\left(-2 - V_t\right)^2$$

$$I_{D2} = 450 = \frac{1}{2}\mu_p\frac{W}{L}\left(-3 - V_t\right)^2$$

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{50}{450} = \frac{\left(2 + V_t\right)^2}{\left(3 + V_t\right)^2}$$

$$\Rightarrow V_t = -1.5\,\text{V} \Rightarrow \mu_p C_{ox}\frac{W}{L} = 400\,\mu\text{A/V}^2$$

Case c).
transistor 3: $V_{GS} = -2\,\text{V}$, $V_{DS} = -1\,\text{V}$. This is a PMOS and it can be either in saturation region or triode region. We assume it is in saturation region:

$$I_{D1} = 200 = \frac{1}{2}\mu_p\frac{W}{L}\left(-2 - V_t\right)^2$$

$$I_{D2} = 800 = \frac{1}{2}\mu_p\frac{W}{L}\left(-3 - V_t\right)^2$$

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{200}{800} = \frac{\left(2 + V_t\right)^2}{\left(3 + V_t\right)^2}$$

$$\Rightarrow V_t = -1\,\text{V} \Rightarrow \mu_p C_{ox}\frac{W}{L} = 400\,\mu\text{A/V}^2$$

The assumption is right, therefore we have:

$$V_{DS} = -1\,\text{V},\ V_{GS} - V_t = -2 + 1 = -1\,\text{V}$$

$$\Rightarrow V_{DS} = V_{GS} - V_t, \text{ edge of saturation}$$

Case d).

transistor 4: $V_{GS} = 2\,\text{V}$, $V_{DS} = 2\,\text{V}$. This is an NMOS in saturation region.

$$I_{D1} = 72 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_t\right)^2$$

When $V_{GS} = 4\,\text{V}$, $V_{DS} = 1\,\text{V}$, this device is in triode region (considering that $V_t < 2\,\text{V}$, we have $V_{DS} < V_{GS} - V_t$).

$$I_{D2} = 270 = \mu_n C_{ox}\frac{W}{L}\left[(4 - V_t) \times 1 - \frac{1}{2} \times 1\right] == \mu_n C_{ox}\frac{W}{L}\left(3.5 - V_t\right)$$

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{72}{270} = \frac{\left(2 - V_t\right)^2}{2\left(3.5 - V_t\right)}$$

$$\Rightarrow V_t = 0.8\,\text{V} \Rightarrow \mu_n C_{ox}\frac{W}{L} = 100\,\mu\text{A/V}^2$$

Problem 4.33

$$I_D = \frac{1}{2}k_n'\frac{W}{L}\left(V_{GS} - V_t\right)^2 \Rightarrow k_n'\frac{W}{L} = 1\text{mA/V}^2$$

a).

$$V_1 = V_{DS} = 3\,\text{V}$$

b).

$$V_2 = V_S = V_D - V_{DS} = 1 - 3 = -2\,\text{V}$$

c).

$$V_3 = V_S = V_D - V_{DS} = 0 - (-3) = 3\,\text{V}$$

d).

$$V_4 = V_D = V_S + V_{DS} = 5 - 3 = 2\,\text{V}$$

When calculating R_{Dmax}, V_{DS} has to be equal to $V_{GS} - V_t$, so that the device is operating on the edge of saturation: $|V_{DS}| = 3 - 1 = 2\,\text{V}$.

a).

$$R_{Dmax} = \frac{3 - 2}{2\,\text{mA}} = 0.5\,\text{k}\Omega$$

b).

$$V_2 = -2\,\text{V} \Rightarrow V_D = -2 + 3 = 1\,\text{V} \Rightarrow R_{Dmax} = \frac{1}{2\,\text{mA}} = 0.5\,\text{k}\Omega$$

c).

$V_2 = -3\,\text{V} \Rightarrow V_S = V_3 = 3\,\text{V}$. Now for V_{DS} to be -2V, V_D has to be 1V.

$$R_{Dmax} = \frac{1}{2\,\text{mA}} = 0.5\,\text{k}\Omega$$

d).

$$V_{GS} = -3\,\text{V} \Rightarrow V_G = V_4 = 2\,\text{V}$$

Adding the resistor between V_4 and drain means that V_D has to be $5-2=3\,\text{V}$ and this leaves 1V to voltage drop on the resistor:

$$R_{Dmax} = \frac{1}{2\,\text{mA}} = 0.5\,\text{k}\Omega$$

When calculating R_{Smax}, since the gate doesn't draw any current, the value of the resistor is immaterial. Assuming that the voltage drop across the current source is at least 2V, R_{Smax} can be calculated as:

a).

$$V_1 = 8\,\text{V},\, V_{GS} = 3\,\text{V} \Rightarrow V_S = 8-3 = 5\,\text{V}$$

$$\Rightarrow R_{Smax} = \frac{5}{2} = 2.5\,\text{k}\Omega$$

b).

$$V_2 = -9+2 = -7\,\text{V},\, V_S = 1-|V_{GS}| = -2\,\text{V}$$

$$\Rightarrow R_{Smax} = \frac{-2-(-7)}{2} = 2.5\,\text{k}\Omega$$

c).

$$V_3 = 10-2 = 8\,\text{V},\, V_S = 0+|V_{GS}| = 3\,\text{V}$$

$$\Rightarrow R_{Smax} = \frac{8-3}{2} = 2.5\,\text{k}\Omega$$

d).

$$V_4 = -5+2 = -3\,\text{V},\, V_S = -3+|V_{GS}| = 0\,\text{V}$$

$$\Rightarrow R_{Smax} = \frac{5}{2} = 2.5\,\text{k}\Omega$$