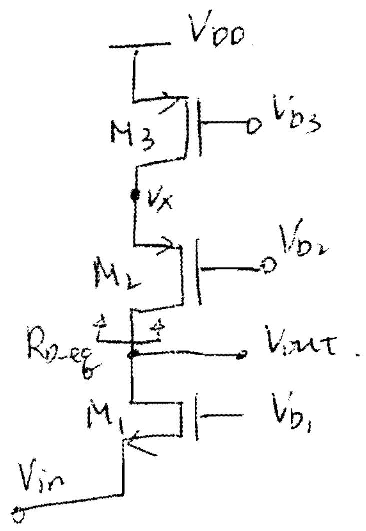
5.21

a) $\gamma = 0$, no body effect.



M_1 is common gate (without R_S)

$$A_V = \frac{(g_m r_o + 1) R_D}{r_o + R_D}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{(g_{m1} r_{o1} + 1) R_{D\text{-}eq}}{r_{o1} + R_{D\text{-}eq}}$$

$$R_{D\text{-}eq} = r_{o2} + (1 + g_{m2} r_{o2}) r_{o3}$$

So $$A_V = \frac{(1 + g_{m1} r_{o1})(r_{o2} + (1 + g_{m2} r_{o2}) r_{o3})}{r_{o1} + r_{o2} + (1 + g_{m2} r_{o2}) r_{o3}}$$

---

by KCL, KVL.

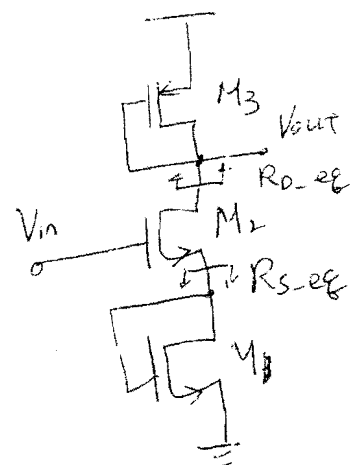
$$\underset{①}{-\frac{V_x}{r_{o3}}} = \underset{②}{g_{m2} V_x + \frac{V_x - V_{out}}{r_{o2}}} = \underset{③}{\frac{V_{out} - V_{in}}{r_{o1}} - g_{m1} V_{in}}$$

$$①, ② \rightarrow V_x = \frac{V_{out}}{1 + r_{o2}(g_{m2} + \frac{1}{r_{o3}})}$$

$$①, ③ \rightarrow \frac{V_{out} - V_{in}}{r_{o1}} - g_{m1} V_{in} = -\frac{V_{out}}{r_{o3} + r_{o2}(1 + g_{m2} r_{o3})}$$

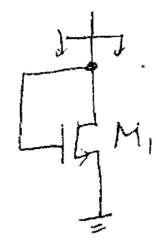
$$A_V = \frac{V_{out}}{V_{in}} = \frac{(1 + g_{m1} r_{o1})(r_{o3} + (1 + g_{m2} r_{o3}) r_{o2})}{r_{o1} + r_{o3} + r_{o2}(1 + g_{m2} r_{o3})}$$

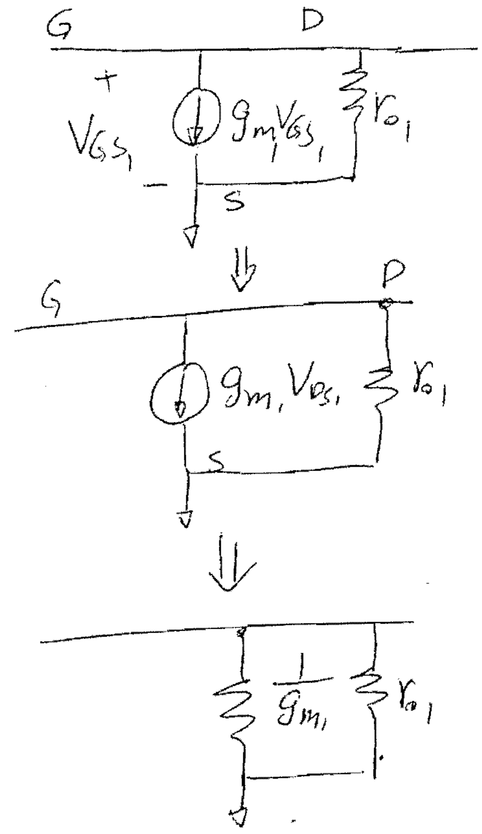
b) $\gamma = 0$ No Body effect.



M_2 : CS with source degenation

$$G_m = \frac{g_m r_o}{R_S + (1+g_m R_S) r_o}$$





$V_{DS_1} = V_{OUT}$.

$R_{S\text{-}eq} = \frac{1}{g_{m_1}} // r_{o_1}$

$$\Rightarrow \quad R_{D\text{-}eq} = \frac{1}{g_{m_3}} // r_{o3}$$

$$R_{S\text{-}eq} = \frac{1}{g_{m_1}} // r_{o_1}$$

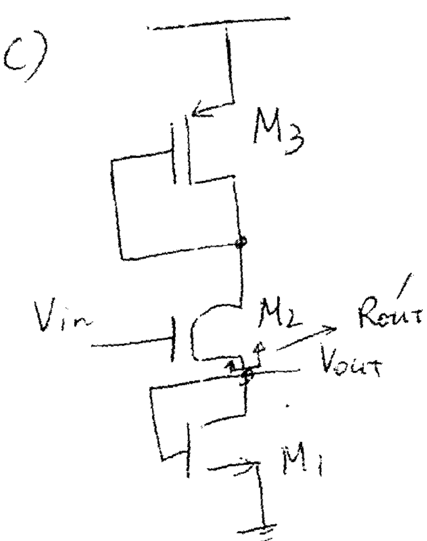
$$G_m = \frac{g_{m_2} r_{o2}}{(\frac{1}{g_{m_1}} // r_{o_1}) + (1 + g_{m_2}(\frac{1}{g_{m_1}} // r_{o_1})) r_{o_2}}$$

$$R_{OUT} = R_{D\text{-}eq} // \{ (1 + g_{m_2} R_{S\text{-}eq}) r_{o_2} + R_{S\text{-}eq}) \}$$

$$R_{out} = \left(\frac{1}{g_{m3}} // r_{o3}\right) // \left\{\left[1 + g_{m2}\left(\frac{1}{g_{m1}} // r_{o1}\right)\right] r_{o2} + \left(\frac{1}{g_{m1}} // r_{o1}\right)\right\}$$

$$A_V = -G_m R_{out} = \cdots$$

c)



M_2 is common drain with $R_{S\text{-}eq}$, $R_{D\text{-}eq}$.

$$R_{S\text{-}eq} = \frac{1}{g_{m1}} // r_{o1}$$

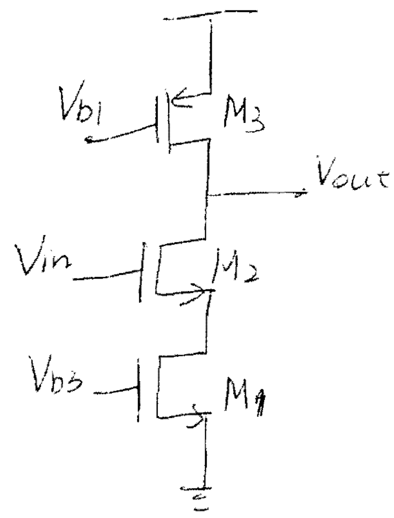
$$R_{D\text{-}eq} = \frac{1}{g_{m3}} // r_{o3}$$

$$A_V = \frac{R_{S\text{-}eq}}{R_{S\text{-}eq} + R_{out}'}$$

$$R_{out}' = \frac{R_{D\text{-}eq} + r_{o2}}{1 + g_{m2} r_{o2}}$$

$$\Rightarrow A_V = \frac{\frac{1}{g_{m1}} // r_{o1}}{\frac{1}{g_{m1}} // r_{o1} + \frac{\frac{1}{g_{m3}} // r_{o3} + r_{o2}}{1 + g_{m2} r_{o2}}}$$

M_2 CS with source degeneration

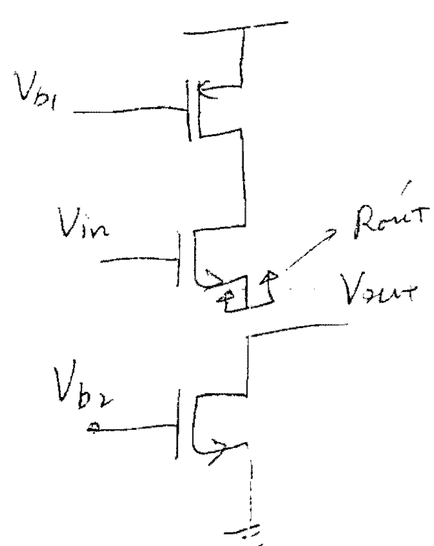


d)

$$G_m = \frac{g_{m2} r_{o2}}{R_{S\_eq} + (1+g_{m2} R_{S\_eq}) r_{o2}} = \frac{g_{m2} r_{o2}}{r_{o1} + (1+g_{m2} r_{o1}) r_{o2}}$$

$$R_{out} = r_{o3} \,//\, [(1+g_{m2} r_{o1}) r_{o2} + r_{o1}]$$

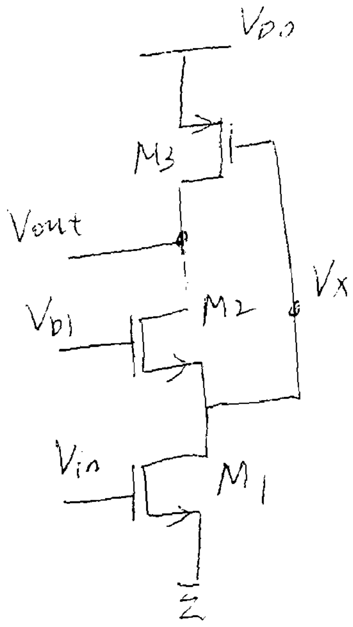
$$A_v = \frac{V_{out}}{V_{in}} = -G_m R_{out} = -\frac{g_{m2} r_{o2} r_{o3}}{r_{o3} + (1+g_{m2} r_{o1}) r_{o2} + r_{o1}}$$



$$R_{out}' = \frac{r_{o3} + r_{o2}}{1 + g_{m2} r_{o2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{r_{o1}}{r_{o1} + R_{out}'} = \frac{r_{o1}(1+g_{m2} r_{o2})}{r_{o1}(1+g_{m2} r_{o2}) + r_{o2} + r_{o3}}$$

(e)



①　　　　②

$$-\left(\frac{V_{DD}}{r_{o3}} + g_{m_3} V_x\right) = \left(\frac{V_{out} - V_x}{r_{o2}} - g_{m_2} V_x\right)$$

③

$$= \frac{V_x}{r_{o1}} + g_{m_1} V_{in}$$

①, ② $\longrightarrow$ $\frac{V_x}{r_{o2}} + g_{m_2} V_x - g_{m_3} V_x = \frac{V_{out}}{r_{o2}} + \frac{V_{out}}{r_{o3}}$

$$\longrightarrow V_x = \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m_2} - g_{m_3}} V_{out}$$

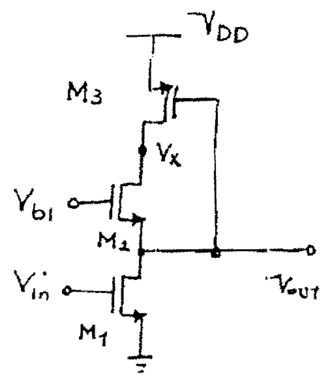
①, ③ $-\frac{V_{out}}{r_{o3}} - g_{m_3} V_x = \frac{V_x}{r_{o1}} + g_{m_1} V_{in}$

$$-\frac{V_{out}}{r_{o3}} - \left(g_{m_3} + \frac{1}{r_{o1}}\right) \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m_2} - g_{m_3}} V_{out} = g_{m_1} V_{in}$$

$\Rightarrow$

$$-V_{out}\left[\frac{1}{r_{o3}} + \frac{(1+g_{m3} r_{o1})(r_{o3}+r_{o2})}{r_{o1} r_{o3}\left[1+(g_{m2}-g_{m3}) r_{o2}\right]}\right] = g_{m1} \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} r_{o1} r_{o3}\left[1+(g_{m2}-g_{m3}) r_{o2}\right]}{r_{o1}\left[1+(g_{m2}-g_{m3}) r_{o2}\right] + (1+g_{m3}\cdot r_{o1})(r_{o3}+r_{o2})}$$



(g)

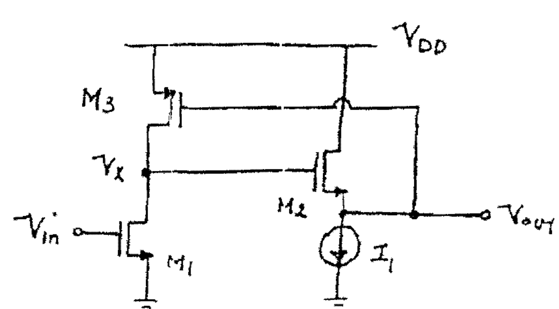
$$V_X = \frac{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}} \cdot V_{out}$$

$$-\frac{V_X}{r_{o3}} - g_{m3} V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1} \cdot V_{in}$$

$$-\frac{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}{\frac{r_{o3}}{r_{o2}} + 1} V_{out} - g_{m3} V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1} V_{in}$$

$$-V_{out}\left[\frac{1+(g_{m2}-g_{m3}) r_{o2}}{r_{o3}+r_{o2}} + g_{m3} + \frac{1}{r_{o1}}\right] = g_{m1} V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} r_{o1} (r_{o2}+r_{o3})}{r_{o1}\left[1+(g_{m2}-g_{m3}) r_{o2}\right] + (r_{o2}+r_{o3})(1+g_{m3}\cdot r_{o1})}$$



(h)

$$-\left(\frac{V_X}{r_{o3}} + \overset{\textcircled{1}}{g_{m3}} V_{out}\right) = g_{m1} \overset{\textcircled{2}}{V_{in}} + \frac{V_X}{r_{o1}}$$

$$\overset{\textcircled{3}}{-\frac{V_{out}}{r_{o2}} + g_{m2}(V_X - V_{out}) = 0} \quad \text{@ output node}$$

$$\textcircled{3} \rightarrow \quad V_X = \frac{\frac{1}{r_{o2}} + g_{m2}}{g_{m2}} \cdot V_{out} = \frac{1+g_{m2} r_{o2}}{g_{m2} r_{o2}} V_{out}$$

$$\textcircled{1}, \textcircled{2} \rightarrow \quad -\left[\left(\frac{1}{r_{o3}} + \frac{1}{r_{o1}}\right)\frac{1+g_{m2}\cdot r_{o2}}{g_{m2}\cdot r_{o2}} + g_{m3}\right] V_{out} = g_{m1}\cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}\cdot g_{m2} r_{o1} r_{o2} r_{o3}}{(r_{o1}+r_{o3})(1+g_{m2}\cdot r_{o2}) + g_{m2} g_{m3} r_{o1} r_{o2} r_{o3}}$$

Dr Wang's cheat sheet.

$$V_{th} = V_{th0} + \gamma\left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right)$$

$$V_{DSAT} = \left(\frac{2I_{DS}}{K'\frac{W}{L}}\right)^{\frac{1}{2}}$$

$$\chi = \frac{\gamma}{\sqrt{2(2\phi_f + V_{SB})}}$$

$$r_o = \frac{1}{\lambda I_{DS}}$$

$$g_m = \sqrt{2K'\frac{W}{L} I_{DS}}$$

$$g_{mb} = \chi g_m$$

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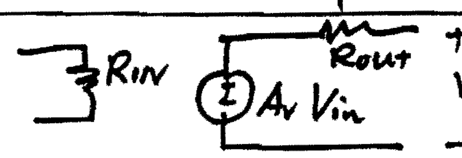
two port

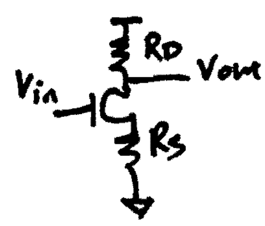


$$R_{in} = \frac{V_{in}}{i_{in}}\Big|_{i_{out}=0} \text{ (open @ output)}$$

$$R_{out} = \frac{V_{out}}{-i_{out}}\Big|_{V_{in}=0} \text{ (short @ input)}$$

$$G_m = \frac{i_{out}}{V_{in}}\Big|_{V_{out}=0} \text{ (short @ output)}, \quad A_v = G_m R_{out}$$

CS with R_S



Accurate .. No Body effect

$$G_m = \frac{-g_m r_o}{R_S + (1+g_m R_S) r_o} \qquad R_{IN} = \infty$$

$$R_{out} = R_D // \left((1+g_m R_S) r_o + R_S\right)$$

$$A_v = G_m R_{out}$$

$$\boxed{\text{If Body effect: } g_m(1+\chi)}$$

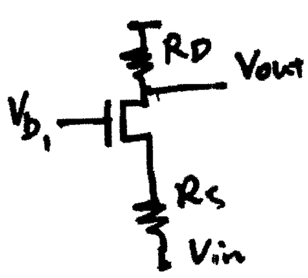
Approximate $g_m R_S \gg 1$, $R_S \ll r_o$

$$G_m \doteq \frac{-g_m}{1+g_m R_S} \quad \text{if } R_S = 0 \text{ High Gain}$$

$$R_{out} \doteq R_D // g_m r_o R_S = R_D$$

$$A_v \doteq -\frac{R_D}{R_S}, \quad R_{in} = \infty$$

CG with R_S



Accurate

$$R_{in} = \frac{R_D + r_o}{g_m r_o + 1}$$

$$R_{out}' = (1+g_m r_o) R_S + r_o = R_S + (1+g_m R_S) r_o$$

$$R_{out} = R_{out}' // R_D$$

$$G_m = \frac{(1+g_m r_o) R_D}{(r_o + R_D)\left[(1+g_m r_o) R_S + r_o\right]}$$

$$A_v = \frac{(g_m r_o + 1) R_D}{r_o + R_D}$$

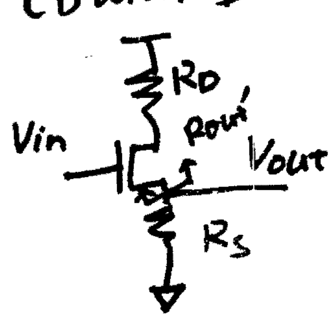
$$R_{in} = \begin{cases} \frac{1}{g_m} & r_o \gg R_D \\ \frac{2}{g_m} & r_o = R_D \\ \text{same as accurate} & R_D \gg r_o \end{cases}$$

$$R_{out} \doteq g_m r_o R_S // R_D$$

$$G_m \doteq g_m \quad R_S \to 0$$

$$A_v \doteq g_m (r_o // R_D) \quad R_S \to 0.$$

CD with R_D



$$R_{IN} = \infty$$

$$R_{out} = R_{out}' // R_S \qquad G_m = \text{too long!}$$

$$R_{out}' = \frac{R_D + r_o}{1 + g_m r_o}$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{R_S}{R_S + R_{out}'}$$

$$A_v = \frac{R_S}{\ldots}$$

$$R_{IN} = \infty$$

$$R_{out}' \doteq \frac{1}{g_m}, \quad R_{out} = \frac{1}{g_m} // R_S$$

G_m = too long!

$$A_v \doteq \frac{g_m R_S}{g_m R_S + 1} \doteq 1.$$

small R_{out}, unit gain