

ECE352 – Lecture Notes
P-N Junction with Bias

$$x_d = \sqrt{\frac{2\epsilon}{q} \left(\frac{1}{N_d^+} + \frac{1}{N_a^-} \right) (V_{bi} - V_a)}$$

Qualitative Description Based on Continuity Equations

Depletion region $(-x'_p < x < x'_n)$

$$\underbrace{\frac{\partial n}{\partial t}}_{=0} = \frac{1}{q} \frac{\partial}{\partial x} (j_n) + \underbrace{(G_n - R_n)}_{=0} \Rightarrow \frac{\partial}{\partial x} (j_n) = 0 \Rightarrow j_n = \text{const}$$

$$\underbrace{\frac{\partial p}{\partial t}}_{=0} = -\frac{1}{q} \frac{\partial}{\partial x} (j_p) + \underbrace{(G_p - R_p)}_{=0} \Rightarrow \frac{\partial}{\partial x} (j_p) = 0 \Rightarrow j_p = \text{const}$$

Consequently we can write

$$j = j_{n_p} (-x'_p) + j_{p_n} (x'_n)$$

In the neutral regions $\left[x \notin (-x'_p, x'_n) \right]$

$$\underbrace{\frac{\partial n}{\partial t}}_{=0} = \frac{1}{q} \frac{\partial}{\partial x} (j_n) + \left(\underbrace{G_n}_{=0} - R_n \right)$$

$$\underbrace{\frac{\partial p}{\partial t}}_{=0} = -\frac{1}{q} \frac{\partial}{\partial x} (j_p) + \left(\underbrace{G_p}_{=0} - R_p \right)$$

Injected minorities recombine with the majorities:

$$R_n = R_p \quad .$$

Therefore, for the steady state we get

$$0 = \frac{1}{q} \frac{\partial}{\partial x} (j_n) - R_n \quad \text{and} \quad 0 = -\frac{1}{q} \frac{\partial}{\partial x} (j_p) - R_p \quad .$$

Subtraction with $(R_n = R_p)$ yields

$$\frac{1}{q} \frac{\partial}{\partial x} (j_n) + \frac{1}{q} \frac{\partial}{\partial x} (j_p) = 0 \quad \text{or else}$$

$$\frac{1}{q} \frac{\partial}{\partial x} (j_n + j_p) = 0 \quad \Rightarrow \quad j_n + j_p = \text{const}$$

Conclusion:

Because the current must be continuous we can use the sum of minority carriers at suitable boundaries to obtain the total current.

Minority carrier concentrations at the depletion boundaries

We use the formula $\frac{n(x_2)}{n(x_1)} = e^{[\psi(x_2) - \psi(x_1)]/V_t}$ with

$x_2 = x'_n$ and $x_1 = -x'_p$ to obtain

$$n(x'_n) = n(-x'_p) e^{\overbrace{[\psi(x'_n) - \psi(-x'_p)]}^{V_{bi} - V_a}/V_t} \quad (*)$$

or else

$$n(x'_n) e^{-(V_{bi} - V_a)/V_t} = n(-x'_p)$$

Note that with $V_a = 0$ we obtain $\underbrace{n(x_n)}_{n_{n_0}} e^{-V_{bi}/V_t} = \underbrace{n(-x_p)}_{n_{p_0}}$

Thus: $n_{p_0} = n_{n_0} e^{-V_{bi}/V_t}$.

The equation (*) can be written as

$$n(x'_n) e^{-V_{bi}/V_t} e^{V_a/V_t} = n(-x'_p)$$

Now (**v. important !**): assuming LLI we have

$$n(x'_n) = n_{n_0}$$

Consequently using the assumption of LLI we obtain

$$n(-x'_p) = n_{p_o} e^{V_a/V_t} .$$

This is called **the Law of the Junction**.

Exercise: Using the LLI assumption, i. e.

$$p(-x'_p) = p(-x_p) = p_{p_o}$$

Derive the Law of the Junction for the holes

$$p(x'_n) = p_{n_o} e^{V_a/V_t} .$$