P-N Junction with Bias – 2

Minority carrier relations in the neutral regions $\left[x \notin \left(-x_p', x_n'\right)\right]$

Ass.: Steady State (DC bias), no field ($\xi = 0$)

Continuity equations for *P*-side (electrons) $x < -x_p'$

$$\underbrace{\frac{\partial n_p}{\partial t}}_{=0} = \frac{1}{q}\frac{\partial}{\partial x}\left(j_{n_p}\right) + \left(\underbrace{G_n}_{=0} - R_n\right)$$

with

$$j_{n_p} = q\mu_n n_p \underbrace{\xi}_{=0} + qD_n \frac{\partial n_p}{\partial x} \qquad ; \qquad R_n = \frac{n_p - n_{p_o}}{\tau_n} = \frac{\Delta n_p}{\tau_n} \quad .$$

Using the assumptions we obtain

$$0 = \frac{1}{q}\frac{\partial}{\partial x}\left(qD_n \frac{\partial n_p}{\partial x}\right) - \frac{\Delta n_p}{\tau_n}$$

or else because $n_{p_o} = const$ we can write the ODE for the excess electrons (minority carriers)

$$D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} = 0$$

with the boundary conditions:

a) at $x \to -\infty$ $\quad \Delta n_p = 0$ $\quad$ because of negligible influence of junction and

b) at $x = -x_p^{'}$ the density is determined by the law of the junction:

$$n\left(-x_p^{'}\right) = n_{p_o} e^{V_a/V_t} \quad .$$

The boundary for the increment, $\Delta n_p\left(x\right) = n\left(x\right) - n_{p_o}$, of excess carriers is therefore

$$\Delta n_p\left(-x_p^{'}\right) = n_{p_o}\left(e^{V_a/V_t} - 1\right).$$

Using these boundary conditions we solve the equation for the increment in the excess carriers and obtain

$$\Delta n_p\left(x\right) = n_{p_o}\left(e^{V_a/V_t} - 1\right) e^{\left(x + x_p^{'}\right)/L_n} \quad ; \quad L_n = \sqrt{D_n \tau_n}$$

This equation is valid for $x \le -x_p^{'}$. *Please work out the details of the solution.*

Exercise:

For $x \ge x_n^{'}$ the derive following solution

$$\Delta p_n\left(x\right) = p_{n_o}\left(e^{V_a/V_t} - 1\right) e^{-\left(x - x_n^{'}\right)/L_p} \quad ; \quad L_p = \sqrt{D_p \tau_p}$$

The Current Densities

The current density in the neutral region on the p-side ($x \le -x_p^{'}$) is

$$j_{n_p} = q\mu_n n_p \underbrace{\xi}_{=0} + qD_n \frac{\partial n_p}{\partial x}$$

For the steady state $\frac{\partial}{\partial t}\left(n_p\right) = 0$ and with the zero electric field we write

$j_{n_p} = qD_n \frac{dn_p}{dx}$ and considering that $n_{p_o} = const \ne \varphi\left(x\right)$ we can write

$$\frac{dn_p}{dx} = \frac{d(\Delta n_p)}{dx}$$

and thus we have

$$j_{n_p}\left(x\right) = qD_n \frac{1}{L_n} n_{p_o}\left(e^{V_a/V_t} - 1\right) e^{\left(x + x_p^{'}\right)/L_n} \quad ; \quad x \le -x_p^{'}$$

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In particular

$$j_{n_p}\left(-x_p^{'}\right) = q\frac{D_n}{L_n} n_{p_o}\left(e^{V_a/V_t} - 1\right) .$$

<u>Exercise</u>

Following the presented approach derive

$$j_{n_p}(x) = q\frac{D_p}{L_p}p_{n_o}\left(e^{V_a/V_t}-1\right)e^{-\left(x-x_n^{'}\right)/L_p} \qquad ; \quad x \geq x_n^{'}$$

and

$$j_{p_n}\left(x_n^{'}\right) = q\frac{D_p}{L_p}p_{n_o}\left(e^{V_a/V_t}-1\right)$$

The total current density

Considering the discussed above approximations (abrupt junction, depletion, and LLI) we know that the current density is constant (DCD bias) and can write

$$j(x) = j_{n_p}\left(-x_p^{'}\right) + j_{p_n}\left(x_n^{'}\right)$$

and finally using the previously derived expressions

$$j(x) = q\left(\frac{D_n}{L_n} n_{p_o} + \frac{D_p}{L_p} p_{n_o}\right)\left(e^{V_a/V_t} - 1\right)$$

Noting that

$$n_{p_o} = \frac{n_i^2}{N_a^-} \qquad \text{and} \qquad p_{n_o} = \frac{n_i^2}{N_d^+}$$

we write

$$j(x) = \underbrace{q\left(\frac{D_n}{L_n}\frac{1}{N_a^-} + \frac{D_p}{L_p}\frac{1}{N_d^+}\right) n_i^2}_{j_s}\left(e^{V_a/V_t} - 1\right)$$

The symbol, j_s , represents the saturation current

$$j_s = q\left(\frac{D_n}{L_n}\frac{1}{N_a^-} + \frac{D_p}{L_p}\frac{1}{N_d^+}\right)n_i^2$$

The equation for the DC current density in the *P*-*N* structure (idealized) is thus

$$j = j_s\left(e^{V_a/V_t} - 1\right) \quad .$$

The current through a device with the cross-sectional area , A, is

$$I = Aj_s\left(e^{V_a/V_t} - 1\right)$$

or else

$$I = I_s\left(e^{V_a/V_t} - 1\right)$$

where the saturation current, I_s , is

$$I_s = q\left(\frac{D_n}{L_n}\frac{1}{N_a^-} + \frac{D_p}{L_p}\frac{1}{N_d^+}\right)n_i^2 A \quad .$$