Dynamic Modeling of *P-N* Junction

I. Abrupt junction

Space Charge (depletion charge)

$$Q_{sc} = qAx_n N_d^+ \qquad x_n = \sqrt{\frac{2\in}{qN_d^+}\frac{N_a^-}{N_a^- + N_d^+}V_{bi}}$$

Therefore

$$Q_{sc} = A\sqrt{2q\in\frac{N_a^- N_d^+}{N_a^- + N_d^+}}\sqrt{V_{bi}}$$ **this is the zero bias charge**

With the bias

$$Q_{sc} = \underbrace{A\sqrt{2q\in\frac{N_a^- N_d^+}{N_a^- + N_d^+}}}_{B}\sqrt{V_{bi} - V_a}$$

or else

$$Q_{sc} = B\sqrt{V_{bi} - V_a}$$

The incremental capacitance

$$C_j = \left|\frac{dQ_{sc}}{dV_a}\right| = B/\sqrt{V_{bi}}\frac{1}{\sqrt{1-\frac{V_a}{V_{bi}}}}$$

Note: $B/\sqrt{V_{bi}} = \frac{Q_{sc}}{V_{bi}} = A\sqrt{2q\in\frac{N_a^- N_d^+}{N_a^- + N_d^+}}\frac{1}{\sqrt{V_{bi}}} = C_{j0}$ **this is the zero bias cap.**

Thus

$$C_j = C_{j0}\frac{1}{\sqrt{1-\frac{V_a}{V_{bi}}}}\ .$$

II. Linearly graded junction

$$Q_{sc} = \frac{1}{2}\frac{W}{2}qa\frac{W}{2}A = \frac{qa}{8}AW^2$$

Electrostatic analysis yields

$$V_{bi} = \frac{qaW^3}{12\in} \qquad \Rightarrow \qquad W = \left(\frac{12\in}{qa}V_{bi}\right)^{\frac{1}{3}}$$

Thus

$$Q_{sc} = A\frac{qa}{8}\left(\frac{12\in}{qa}V_{bi}\right)^{\frac{2}{3}}$$

With the bias

$$Q_{sc} = A\frac{qa}{8}\left[\frac{12\in}{qa}\left(V_{bi} - V_a\right)\right]^{\frac{2}{3}}$$

or else

$$Q_{sc} = \underbrace{A\frac{qa}{8}\left(\frac{12\in V_{bi}}{qa}\right)^{\frac{2}{3}}}_{Q_{sc0}}\left(1 - \frac{V_a}{V_{bi}}\right)^{\frac{2}{3}}$$

The incremental capacitance

$$C_j = \left|\frac{dQ_{sc}}{dV_a}\right| = \frac{2}{3}\underbrace{A\frac{qa}{8}\left(\frac{12\in V_{bi}}{qa}\right)^{\frac{2}{3}}\frac{1}{V_{bi}}}_{C_{j0}}\left(1 - \frac{V_a}{V_{bi}}\right)^{-\frac{1}{3}}$$

$$C_j = C_{j0}\left(1 - \frac{V_a}{V_{bi}}\right)^{-\frac{1}{3}}$$

Considering both cases I. and II. we can write a general form for the junction capacitance:

$$C_j = C_{j0}\left(1 - \frac{V_a}{V_{bi}}\right)^{-m} \quad ; \quad \frac{1}{3} \le m \le \frac{1}{3}$$

Where *m* is the grading coefficient.

Diffusion Capacitance

Excess charges

***p*-region**

$$Q_n = qA\int_{-\infty}^{-x_p'} \Delta n_p(x)dx$$

***n*-region**

$$Q_p = qA\int_{x_n}^{\infty} \Delta p_n(x)dx$$

Total diffusion charge

$$Q_{S_p} = Q_p + Q_n$$

The diffusion capacitance

$$C_{S_p} = \left|\frac{dQ_{S_p}}{dV_a}\right| \cong \frac{\tau_p}{V_t}\underbrace{I_s\left(e^{V_a/V_t} - 1\right)}_{I_d}$$

or else

$$C_{S_p} \cong \frac{\tau_p}{V_t} I_d \quad .$$