

Lecture 10B Espresso Representations and Basic Operations

Function Definition

- Assume you have a logic function with n -input variables and m -output variables

Let $B = \{0, 1\}$ // input alphabet is 1 or 0
 $Y = \{0, 1, 2\}$ // output alphabet is 1, 0, or 2 (don't care)

- Logic function is simply a mapping of input combinations to output values

$F : B^n \rightarrow Y^m$ where $x = [x_1, \dots, x_n] \in B^n$ is the input
 and $y = [y_1, \dots, y_m]$ is the output of F

Function Mapping

Example 1: $F_1(a, b) = a'$; $F_2(a, b) = a + b$

$n = 2$
 $m = 2$

When $m \geq 2$ we have a multiple output function

a	b	F_1	F_2
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

$F : B^n \rightarrow Y^m$

For each input combination we map to an output combination

Example 2: $F_0(a, b, c) = b' + ac$

$n = 3$
 $m = 1$

When $m = 1$ we have a single output function

a	b	c	F_0
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

On, Off, and DC Sets

- For a given function we can define the **on-set** $x^{ON} \subseteq B^n$ as the set of input values x such that $F(x) = 1$

$$F^{ON} = \{ [0, 0, 0], [0, 0, 1], [1, 0, 0], [1, 0, 1], [1, 1, 1] \}$$

- For a given function we can define the **off-set** $x^{OFF} \subseteq B^n$ as the set of input values x such that $F(x) = 0$

$$F^{OFF} = \{ [0, 1, 0], [0, 1, 1], [1, 1, 0] \}$$

- For a given function we can define the **don't care-set** $x^{DC} \subseteq B^n$ as the set of input values x such that $F(x) = 2$

$$F^{DC} = \{ \} \quad // \text{empty}$$

a	b	c	F_x
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

On, Off, and DC Sets

Example 3: Given the following truth table, identify F^{ON} , F^{OFF} , and F^{DC}

$$F^{ON} = \{ [0, 0, 0], [0, 0, 1], [1, 0, 0], [1, 0, 1], [1, 1, 0] \}$$

$$F^{OFF} = \{ [0, 1, 0], [0, 1, 1] \}$$

$$F^{DC} = \{ [1, 1, 1] \}$$

a	b	c	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	2

Completely Specified Functions

- A completely specified function (c.s.f) is a function where all values of the input map to a 1 or 0 (i.e. no don't care conditions)

Ex 1 and 2 are completely specified functions

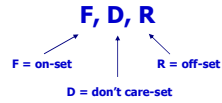
a	b	F_1	F_2
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

Ex 3 is not a completely specified function

a	b	c	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	2

On, Off, and DC Sets in Espresso

- In Espresso you will see the triple F, D, R
 - Typically F and D are provided and R is derived



How to get R?
 $R = (F \cup D)'$

ECE 474a/575a
 Susan Lysdecky

7 of 25

Logic Functions Operations & Definitions

Complement of a c.s.f. (completely specified function) F' is defined as $F^{ON} = F^{OFF}$ and $F^{OFF} = F^{ON}$ (i.e. switch on and off sets)

x1	x2	F	F'
0	0	0	1
0	1	1	0
1	0	0	0
1	1	0	1

Intersect (or product) of two c.s.f. F and G, denoted as $F \cdot G$ or $F \cap G$, is the c.s.f. H where $H^{ON} = F^{ON} \cap G^{ON}$ (i.e. must be in both)

x1	x2	F	G	H
0	0	0	1	0
0	1	1	1	1
1	0	1	0	0
1	1	0	0	0

Difference between two c.s.f. F and G, denoted as $F - G$, is the c.s.f. H where $H^{ON} = F^{ON} \cap G^{ON}$ (i.e. it's in F but not in G)

x1	x2	F	G	G'	H
0	0	0	0	1	0
0	1	1	1	0	0
1	0	1	0	1	1
1	1	1	1	0	0

ECE 474a/575a
 Susan Lysdecky

8 of 25

Logic Functions Operations & Definitions

Union (or sum) of two c.s.f. F and G, denoted as $F + G$ or $F \cup G$, is the c.s.f. where $H^{ON} = F^{ON} \cup G^{ON}$ (i.e. it's in either F or G)

x1	x2	F	G	H
0	0	1	0	1
0	1	0	0	0
1	0	0	1	1
1	1	0	1	1

A **tautology** is a c.s.f. whose off-set is empty, written $F \equiv 1$ (i.e. function always evaluates to 1)

$F \cup D \cup R$ is a tautology

F, D, R are mutually disjoint (no elements in common)

ECE 474a/575a
 Susan Lysdecky

9 of 25

Operations on Cubes

Union (or sum) of two cubes, written as $c \cup d$ or $c+d$, is the set of vertices covered by the input part of either c or d

Basic idea: combine all cubes

$$c_1 = [2 \ 0 \ 2 \ 4 \ 3]$$

$$d_1 = [1 \ 2 \ 0 \ 4 \ 3]$$

$$c_1 \cup d_1 = [2 \ 0 \ 2 \ 4 \ 3] \\ [1 \ 2 \ 0 \ 4 \ 3]$$

ECE 474a/575a
Susan Lyshecky

19 of 25

Operations on Cubes

What about **Complement**?

Use DeMorgan's Law to determine what happens to a product term when complemented

$$(\mathbf{ab})' = \mathbf{a}' + \mathbf{b}'$$

$$F(\mathbf{a}, \mathbf{b}) = \mathbf{a} \quad [1 \ 2]$$

If the cube has a single term we can simply switch the 1 & 0 entry

$$F'(\mathbf{a}, \mathbf{b}) = \mathbf{a}' \quad [0 \ 2]$$

$$F(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \mathbf{ab} \quad [1 \ 1 \ 2]$$

switch every 1 & 0 entry, but we must also make sure each row has only one 1/0 entry

$$F'(\mathbf{a}, \mathbf{b}, \mathbf{c}) = (\mathbf{ab})' \\ = \mathbf{a}' + \mathbf{b}'$$

$$[0 \ 2 \ 2] \\ [2 \ 0 \ 2]$$

ECE 474a/575a
Susan Lyshecky

20 of 25

Operations on Cubes

What about **Complement**?

Use DeMorgan's Law to determine what happens to a product term when complemented

$$(\mathbf{ab})' = \mathbf{a}' + \mathbf{b}'$$

$$F(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \mathbf{a}'\mathbf{bc} \quad [0 \ 1 \ 1]$$

Again switch every 1 & 0 entry, and make sure each row has only one 1/0 entry

$$F'(\mathbf{a}, \mathbf{b}, \mathbf{c}) = (\mathbf{a}'\mathbf{bc})' \\ = \mathbf{a} + \mathbf{b}' + \mathbf{c}'$$

$$[1 \ 2 \ 2] \\ [2 \ 0 \ 2] \\ [2 \ 2 \ 0]$$

Many other transformations exist (distance, consensus, etc..) we'll stick with the basic ones for now

ECE 474a/575a
Susan Lyshecky

21 of 25

Espresso Optimization Goal

- Espresso algorithm returns a "minimized cover"
 - What is the algorithm trying to minimize?

$$\mathcal{F} = (NPT, NLI, NLO)$$

NLO = # of literals in output part

NLI - # of literals (non-2's) in input part of cover

NPT - # of product terms in a cover

ECE 474a/575a
Susan Lyshecky

22 of 25

Espresso Subroutine

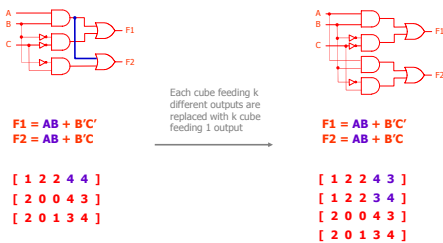
- Many smaller subroutines used in Espresso
 - We will only cover a few
 - Unwrap
 - Unate Complement
 - Complement
 - Expand

ECE 474a/575a
Susan Lyshecky

23 of 25

Unwrap(F)

- Incoming data may have output sharing
 - Apply Unwrap(F) to the input so we start with a less biased starting point
 - When complete algorithm can decide what sharing is desirable



ECE 474a/575a
Susan Lyshecky

24 of 25

Coming Soon ...

- Unate Complement
- Complement
- Expand

ECE 474a/574a
Susan Lysecky

25 of 25
