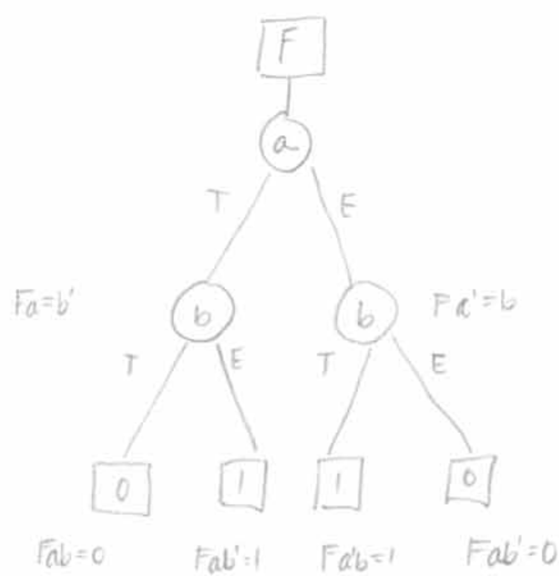
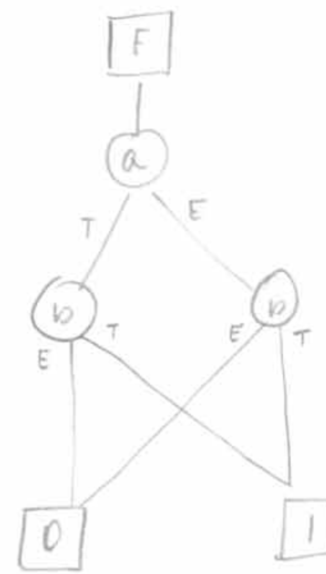
EX1: convert $F = a \text{ xor } b$ into a BDD (ordeiing $a < b$)

$F = a'b + ab'$

$F_a = b'$ $\quad F_{a'} = b$

$F_{ab} = 0$ $\quad F_{ab'} = 1$ $\quad F_{a'b} = 1$ $\quad F_{a'b'} = 0$





You could use multiple terminal nodes in your drawing to make BDD easier to draw but internal structure will have 1 pair of terminal nodes

Ex2: use <u>ITE Algorithm to convert $F=a'b+b'c'$ into BDD</u> (variable ordering $a \le b \le c$)

According to ITE operator table provided $F+G = ITE(F,1,G)$, here $F=a'b$ and $G=b'c'$

$= ITE(a'b, 1, b'c')$

$= ITE(a, ITE((a'b)_a, (1)_a, (b'c')_a), ITE((a'b)_{a'}, (1)_{a'}, (b'c')_a))$

$= ITE(a, ITE(0,1,b'c'), ITE(b,1,b'c'))$ // $ITE(0,1,b'c') = b'c'$

$= ITE(a, b'c', ITE(b,1,b'c'))$ //expand $ITE(b,1,b'c')$ w.r.t. b

$= ITE(a, b'c', ITE(b, ITE((b)_b, (1)_b, (b'c')_b), ITE((b)_{b'}, (1)_{b'}, (b'c')_{b'})))$

$= ITE(a, b'c', ITE(b, ITE(1,1,0), ITE(0,1,c')))$ // $ITE(1,1,0)=1$ and $ITE(0,1,c')=c'$

$= ITE(a, b'c', ITE(b,1,c'))$

// $b'c' = ITE(b',c',0)$ according to table provided.

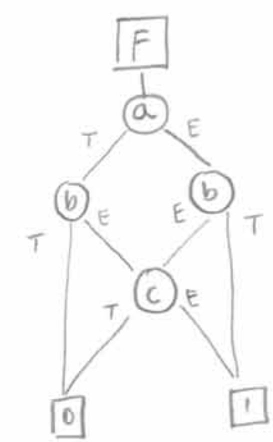
$= ITE(a, ITE(b',c',0), ITE(b,1,c'))$

// cannot have a node labled b' so we expand w.r.t b

$= ITE(a, ITE(b, ITE((b')_b, (c')_b, (0)_b), ITE((b')_{b'}, (c')_{b'}, (0)_{b'})), ITE(b,1,c'))$

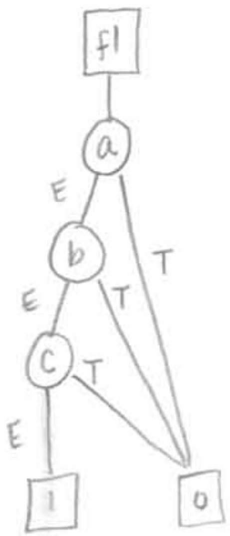
$= ITE(a, ITE(b, ITE(0,c',0), ITE(1,c',0)), ITE(b,1,c'))$

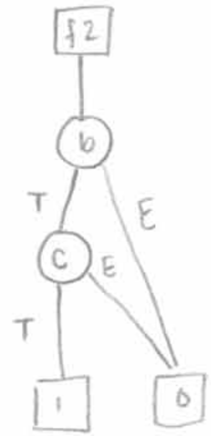
$= ITE(a, ITE(b,0,c'), ITE(b,1,c'))$



ex3: Iterative Approach to building BDDs.

$F = a'b'c' + bc$ $(a \le b \le c)$

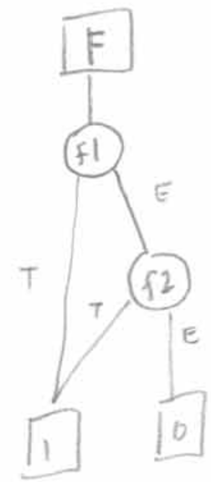




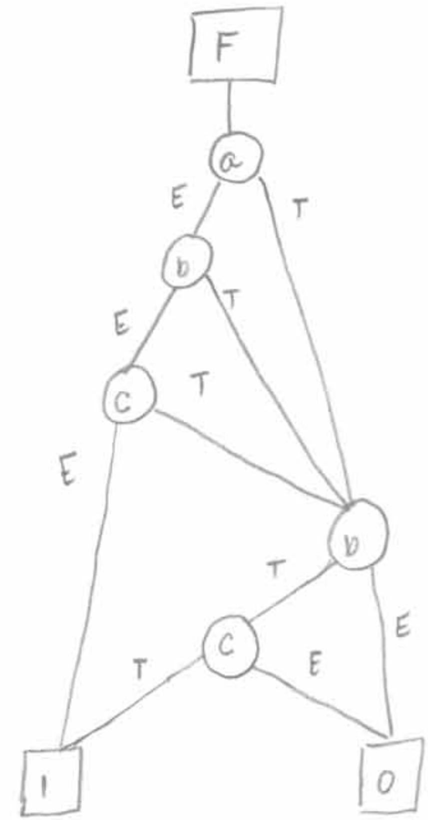
$a'b'c'$

bc

How do I combine them?
$F = f1 + f2$



expand f1 & f2



problem lost variable ordering

no longer reduced.

try again in ex4 with recursive approach.

Ex4: ITE Recursive Approach to BDDs

$F = a'b'c' + bc \qquad (a \le b \le c)$

$= ITE(a'b'c', 1, bc)$

$= ITE(a, ITE((a'b'c')_a, (1)_a, (bc)_a), ITE((a'b'c')_{a'}, (1)_{a'}, (bc)_{a'}))$

$= ITE(a, ITE(0,1,bc), \underbrace{ITE(b'c', 1, bc)})$

$= ITE(a, bc, ITE(b, ITE((b'c')_b, (1)_b, (bc)_b), ITE((b'c')_{b'}, (1)_{b'}, (bc)_{b'})))$

$= ITE(a, bc, ITE(b, ITE(0,1,c), \underbrace{ITE(c', 1, 0)})))$

$= ITE(a, bc, ITE(b, c, ITE(c, ITE(0,1,0), ITE(1,1,0))))$

$= ITE(a, bc, ITE(b, c, ITE(c, 0, 1)))$

$= ITE(a, \underline{\underline{bc}}, ITE(b, c, c'))$

$= ITE(a, ITE(b, c, 0), ITE(b, c, c'))$

✓ ordered
✓ reduced