ECE 474A/57A
Computer-Aided Logic Design

Lecture 13B
Scheduling Algorithms Continued

List Scheduling (LIST_R)

- Considers minimum-resource, latency-constrained scheduling problem

```
LIST_R( G_S(V,E), λ̄ ){
    a = 1;

    Compute the latest possible start times t_i^L by ALAP( G(V, E), λ̄);
    if( t_0^L < 0)
        return (Φ);

    l = 1;

    repeat {
        for each resource type k = 1, 2, …, n_res {
            Determine candidate operations U_k;
            Compute the slacks {s_i = t_i^L - l  ∀ v_i ∈ U_k};
            Schedule the candidate operations with zero slack and update a;
            Schedule the candidate operations requiring no additional resources;
        }
        l = l + 1;
    } until (v_n is scheduled);
    return (t, a);
}
```

Vector a indicates the number of each type of resource available

Algorithm exits if ALAP detects no feasible solution with dedicated resources

Time step

Operations of type k whose predecessors are completed by time l

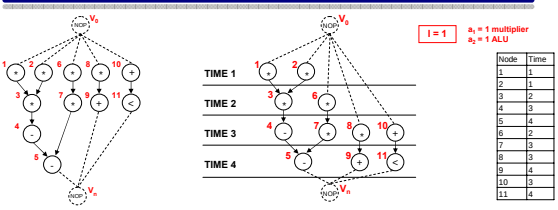
Compute slack of all candidates (current time – ALAP time)

Scheduled any operation with 0 slack to meet timing requirement, add resources if needed

Fill in unused resources by scheduling any available operation

Keep going until we have scheduled the sink node v_n

LIST_R Scheduling
Example 1



l = 1

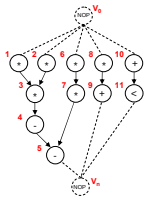
a_1 = 1 multiplier
a_2 = 1 ALU

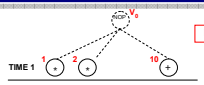
Node	Time
1	1
2	1
3	2
4	3
5	4
6	2
7	3
8	3
9	4
10	3
11	4

- Assume all operations have unit delay, latency of 4 is required
  - Initialize vector a so all entries have value of 1
  - Compute the latest start times of all vectors by using ALAP()
  - Set time step equal to 1

LIST_R Scheduling

Example 1





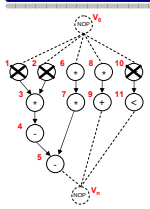
$l = \cancel{2}$ 2 $a_1 = \cancel{2}$ multiplier
$a_2 = 1$ ALU

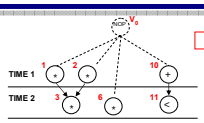
Node	Time
1	1
2	1
3	2
4	3
5	4
6	2
7	3
8	3
9	4
10	3
11	4

	Multipliers	ALUs
Determine candidate operations	$U = \{ v_1, v_2, v_6, v_8 \}$	$U = \{ v_{10} \}$
Compute the slacks	$v_1 = 1-1 = 0$ $v_2 = 1-1 = 0$ $v_6 = 2-1 = 1$ $v_8 = 3-1 = 2$	$v_{10} = 3-1 = 2$
Schedule candidate operations with zero slack and update a	$S = \{ v_1, v_2 \}$, $a_1 = 2$	no zero slack operations
Schedule candidate operations requiring no additional resources	no spare multipliers	$S = \{ v_{10} \}$
Increment time step	$l = 1 + 1 = 2$	
Has v_n been scheduled yet?	No. Repeat loop.	

LIST_R Scheduling

Example 1





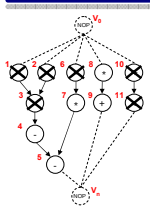
$l = \cancel{3}$ 3 $a_1 = 2$ multiplier
$a_2 = 1$ ALU

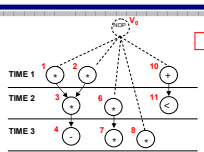
Node	Time
1	1
2	1
3	2
4	3
5	4
6	2
7	3
8	3
9	4
10	3
11	4

	Multipliers	ALUs
Determine candidate operations	$U = \{ v_3, v_6, v_8 \}$	$U = \{ v_{11} \}$
Compute the slacks	$v_3 = 2-2 = 0$ $v_6 = 2-2 = 0$ $v_8 = 3-2 = 1$	$v_{11} = 4-2 = 2$
Schedule candidate operations with zero slack and update a	$S = \{ v_3, v_6 \}$	no zero slack operations
Schedule candidate operations requiring no additional resources	no spare multipliers	$S = \{ v_{11} \}$
Increment time step	$l = 2 + 1 = 3$	
Has v_n been scheduled yet?	No. Repeat loop.	

LIST_R Scheduling

Example 1





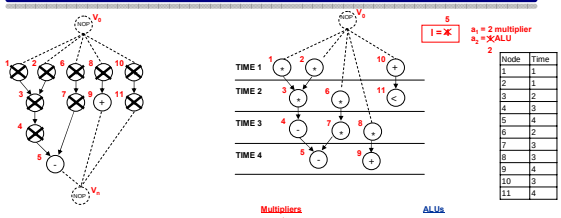
$l = \cancel{4}$ 4 $a_1 = 2$ multiplier
$a_2 = 1$ ALU

Node	Time
1	1
2	1
3	2
4	3
5	4
6	2
7	3
8	3
9	4
10	3
11	4

	Multipliers	ALUs
Determine candidate operations	$U = \{ v_7, v_8 \}$	$U = \{ v_4 \}$
Compute the slacks	$v_7 = 3-3 = 0$ $v_8 = 3-3 = 0$	$v_4 = 3-3 = 0$
Schedule candidate operations with zero slack and update a	$S = \{ v_7, v_8 \}$	$S = \{ v_4 \}$
Schedule candidate operations requiring no additional resources	no spare multipliers	no spare ALUs
Increment time step	$l = 1 + 1 = 4$	
Has v_n been scheduled yet?	No. Repeat loop.	

LIST_R Scheduling

Example 1



I = 5

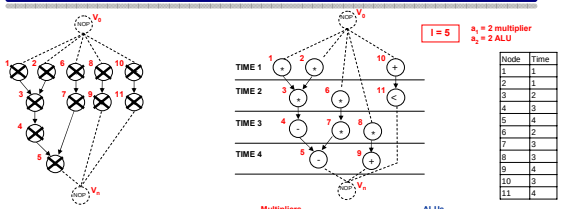
a_1 = 2 multiplier
a_2 = ALU 2

Node	Time
1	1
2	1
3	2
4	3
5	4
6	2
7	3
8	3
9	4
10	3
11	4

	Multipliers	ALUs
Determine candidate operations	U = { ✱ }	U = { v_5, v_9}
Compute the slacks		v_5 = 4-4 = 0 v_9 = 4-4 = 0
Schedule candidate operations with zero slack and update a	S = { ✱ }	S = {v_5, v_9}; a = 2
Schedule candidate operations requiring no additional resources	no multiplier operations	no spare ALUs
Increment time step	I = 4 + 1 = 5	
Has v_n been scheduled yet?	No. Repeat loop.	

LIST_R Scheduling

Example 1



I = 5

a_1 = 2 multiplier
a_2 = 2 ALU

Node	Time
1	1
2	1
3	2
4	3
5	4
6	2
7	3
8	3
9	4
10	3
11	4

	Multipliers	ALUs	
Determine candidate operations	U = { ✱ }	U = { ✱}	U = { V_n}
Compute the slacks			
Schedule candidate operations with zero slack and update a			
Schedule candidate operations requiring no additional resources			
Increment time step			
Has v_n been scheduled yet?	Yes. Done		

Force-Directed Scheduling (FDS)

- Heuristic scheduling algorithms
  - Consider the unscheduled CDFG under a physics-based spring model
  - Operators are subjected to physical 'forces', both repelling and attracting them to particular time slices
    - Larger the force, the larger the concurrency
  - Goal is to find the optimal placement of vertices into a schedule, when subject to these 'forces'

- Minimum latency under resource-constraint
  - Force directed list scheduling
  - Extension of list scheduling algorithms
- Minimum resource under latency-constraint
  - Force directed scheduling

This is the one we will consider

Force-Directed Scheduling (FDS)

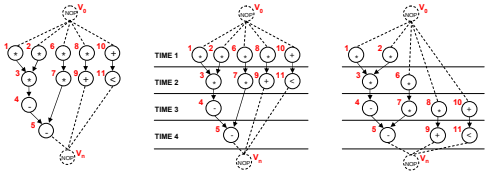
- Force-Directed Scheduling
  - Minimum resource under latency constraint

```
FDS( G(V,E), λ ){
    repeat {
        Compute the time frames;
        Compute the operations and type probabilities;
        Compute the self-forces, predecessor/successor forces and total forces;
        Schedule the operation with least force and update its time-frame;
    } until (all operations scheduled);
    return (t);
}
```


Force-Directed Scheduling (FDS)

Time Frames

- Time frame of an operation is the time interval where it can be scheduled
  - Denoted by $\{[t_i^s, t_i^l]; i = 0, 1, ..., n\}$
  - Earliest and latest start times can be computed by ASAP and ALAP algorithms

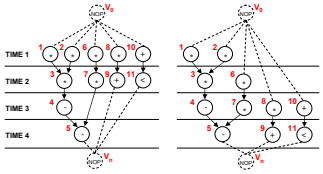


- Width of time frame of an operation is equal to its mobility plus 1

Force-Directed Scheduling (FDS)

Example 2

- Time frames for various operation assuming a latency bound of 4
  - Latency bound needed for ALAP scheduling



operation v_1
ASAP time = 1
ALAP time = 1
time frame = [1, 1]

operation v_2
ASAP time = 1
ALAP time = 1
time frame = [1, 1]

operation v_6
ASAP time = 1
ALAP time = 2
time frame = [1, 2]

operation v_8
ASAP time = 1
ALAP time = 3
time frame = [1, 3]

Force-Directed Scheduling (FDS)

- Force-Directed Scheduling
  - Minimum resource under latency constraint

```
FDS( G(V,E), λ ){
   repeat {
      Compute the time frames;
      Compute the operations and type probabilities;
      Compute the self-forces, predecessor/successor forces and total forces;
      Schedule the operation with least force and update its time-frame;
   } until (all operations scheduled);
   return (t);
}
```


Force-Directed Scheduling (FDS)

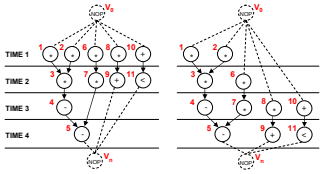
Operation Probability

- Operation Probability is a function
  - Equal to zero outside of the corresponding time frame
  - Equal to reciprocal of the frame width inside the time frame
- Denoted the probability of the operations at time *l* by $\{p_i(l); i = 0, 1, ..., n\}$
- What is the significance?
  - Operations whose time frame is one unit wide are bound to start in one specific time
  - For remaining operations, the larger the width, the lower the probability that the operation is scheduled in any given step inside the corresponding time frame

Force-Directed Scheduling (FDS)

Example 3

- Operation Probability for various operations
  - Equal to zero outside of the corresponding time frame
  - Equal to reciprocal of the frame width inside the time frame



operation v_1	operation v_2	operation v_6	operation v_8
time frame = [1, 1]	time frame = [1, 1]	time frame = [1, 2]	time frame = [1, 3]
frame width = 1	frame width = 1	frame width = 2	frame width = 3
$p_1(1) = 1, p_1(2) = 0$	$p_2(1) = 1, p_2(2) = 0$	$p_6(1) = 0.5, p_6(2) = 0.5$	$p_8(1) = 0.3, p_8(2) = 0.3$
$p_1(3) = 0, p_1(4) = 0$	$p_2(3) = 0, p_2(4) = 0$	$p_6(3) = 0, p_6(4) = 0$	$p_8(3) = 0.3, p_8(4) = 0$

Force-Directed Scheduling (FDS)

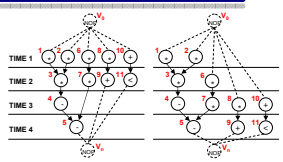
Type Distribution

- Type Distribution is the sum of probabilities of the operations implemented by a specific resource at any time step of interest
  - Denote distribution at time *l* by $\{q_k(l);\ k = 1, 2, ..., n_{res}\}$
- Distribution graph is a plot of any operation-type distribution over the scheduled steps
  - Shows likelihood that a resource is used at each scheduled step
  - Uniform plot in a distribution graph means that a type is evenly scattered in the schedule and a good measure of utilization

Force-Directed Scheduling (FDS)

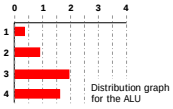
Example 4

- Distribution graph for ALU
  - Sum of probabilities of the operations implemented by a specific resource at any time step of interest



	p(1)	p(2)	p(3)	p(4)
v_4 = [3, 3], width = 1	0	0	1	0
v_5 = [4, 4], width = 1	0	0	0	1
v_9 = [2, 4], width = 3	0	0.3	0.3	0.3
v_{10} = [1, 3], width = 3	0.3	0.3	0.3	0
v_{11} = [2, 4], width = 3	0	0.3	0.3	0.3

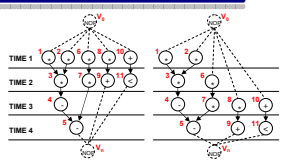
$q_a(1) = 0 + 0 + 0 + 0.3 + 0 = 0.3$
$q_a(2) = 0 + 0 + 0.3 + 0.3 + 0.3 = 0.9$
$q_a(3) = 1 + 0 + 0.3 + 0.3 + 0.3 = 1.9$
$q_a(4) = 0 + 1 + 0.3 + 0 + 0.3 = 1.6$



Force-Directed Scheduling (FDS)

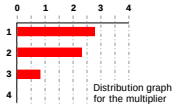
Example 5

- Distribution graph for Multiplier
  - Sum of probabilities of the operations implemented by a specific resource at any time step of interest



	p(1)	p(2)	p(3)	p(4)
v_1 = [1, 1], width = 1	1	0	0	0
v_2 = [1, 1], width = 1	1	0	0	0
v_3 = [2, 2], width = 1	0	1	0	0
v_6 = [1, 2], width = 2	0.5	0.5	0	0
v_7 = [2, 3], width = 2	0	0.5	0.5	0
v_8 = [1, 3], width = 3	0.3	0.3	0.3	0

$q_t(1) = 1 + 1 + 0 + 0.5 + 0 + 0.3 = 2.8$
$q_t(2) = 0 + 0 + 1 + 0.5 + 0.5 + 0.3 = 2.3$
$q_t(3) = 0 + 0 + 0 + 0 + 0.5 + 0.3 = 0.8$
$q_t(4) = 0 + 0 + 0 + 0 + 0 + 0 = 0$



Force-Directed Scheduling (FDS)

- Force-Directed Scheduling
  - Minimum resource under latency constraint

```
FDS( G(V,E), λ ){
   repeat {
      Compute the time frames;
      Compute the operations and type probabilities;
      Compute the self-forces, predecessor/successor forces and total forces;
      Schedule the operation with least force and update its time-frame;
   } until (all operations scheduled);
   return (t);
}
```


Force-Directed Scheduling (FDS)

Self Force

- Self Force
  - Scheduling an operation will effect overall concurrency
  - Every operation has "self force" for every C-step of its time frame
  - Desirable scheduling will have negative self force

Force(i) = DG(i) * x(i)

DG(i) = Current Distribution Graph value
x(i) = Change in operation's probability

$$\text{Self Force}(j) = \sum_{i=t}^{b} \text{Force}(i)$$

Force-Directed Scheduling (FDS)

Example 6

- Calculate Self Force for v_6
  - Assignment of v6 to time step 1
  - Assignment of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Current Distribution Graph value
x(i) = Change in operation's probability

$$\text{Self Force}(j) = \sum_{i=t}^{b} \text{Force}(i)$$

Assuming v6 assigned to time step 1

Self force = 2.8(1-0.5) + 2.3(0-0.5)

Distribution graph values to time step 1 and 2

1 indicates that v6 schedule in time 1, minus the operator probability in time 1

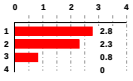
0 indicates that v6 is NOT scheduled in time 1, minus the operator probability in time 2

Time frame and operation probability for v_6

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

Distribution graph for the multiplier



Force-Directed Scheduling (FDS)

Example 6

- Calculate Self Force for v_6
  - Assignment of v6 to time step 1
  - Assignment of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Current Distribution Graph value
x(i) = Change in operation's probability

$$\text{Self Force}(j) = \sum_{i=t}^{b} \text{Force}(i)$$

Assuming v6 assigned to time step 1

Self force = 2.8(1-0.5) + 2.3(0-0.5)
= 0.25

Assuming v6 assigned to time step 2

Self force = 2.8(0-0.5) + 2.3(1-0.5)
= -0.25

Want to reduce force (concurrency), time step 2 looks better

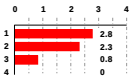
How does this impact other operations?

Time frame and operation probability for v_6

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

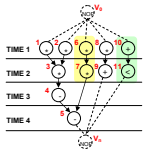
Distribution graph for the multiplier



Force-Directed Scheduling (FDS)

Predecessor/Successor Forces

- Predecessor/Successor Force
  - Scheduling an operation may affect the time frames of other linked operations
  - This may negate the benefits of the desired assignment
  - Predecessor/Successor Forces = Sum of Self Forces of any implicitly scheduled operations



If v_6 scheduled in time 2, then v_7 has to be scheduled in time 3

If v_{11} scheduled in time 3, then v_{10} has to be scheduled in time 1 or 2

Force-Directed Scheduling (FDS)

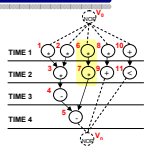
Example 7

- Calculate Predecessor/Successor Force for v_6
  - Assign of v6 to time step 1
  - Assign of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Curr Distrb Graph value
x(i) = Change in op prob

$$\text{Self Force}(j) = \sum_{i=t}^{b} \text{Force}(i)$$



Assuming v6 assigned to time step 1

no predecessor effected

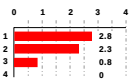
Predecessor force = 0

no successor effected
v_7 can be scheduled at time 2 or 3

Successor force = 0

Total force = Self Force + Predecessor force + Successor force
= 0.25 + 0 + 0
= 0.25

Distribution graph for the multiplier



Time frame and operation probability for v_6 and v_7

v_6 = [1, 2], width = 2

p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

v_7 = [2, 3], width = 2

p(1)=0, p(2)=0.5, p(3)=0.5, p(4)=0

Force-Directed Scheduling (FDS)

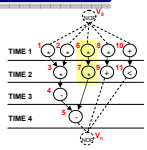
Example 7

- Calculate Predecessor/Successor Force for v_6
  - Assign of v6 to time step 1
  - Assign of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Curr Distrb Graph value
x(i) = Change in op prob

Self Force(j) = $\sum_{i=t}^{b}$ Force(i)



Assuming v6 assigned to time step 2

no predecessor effected

Predecessor force = 0

v_7 can only be scheduled at time 3

Successor force = sum of self forces of implicitly scheduled operations
= 2.3(0-0.5) + 0.8(1-0.5)
= -0.75

Distribution graph for the multiplier

	0	1	2	3	4
1			2.8		
2			2.3		
3			0.8		
4			0		

Total force = Self Force + Predecessor force + Successor force
= -0.25 + 0 + -0.75
= -1

Time frame and operation probability for v_6 and v_7

v_6 = [1, 2], width = 2
p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

v_7 = [2, 3], width = 2
p(1)=0, p(2)=0.5, p(3)=0.5, p(4)=0

Force-Directed Scheduling (FDS)

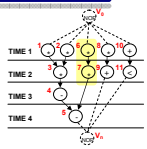
Example 7

- Calculate Predecessor/Successor Force for v_6
  - Assign of v6 to time step 1
  - Assign of v6 to time step 2

Force(i) = DG(i) * x(i)

DG(i) = Curr Distrb Graph value
x(i) = Change in op prob

Self Force(j) = $\sum_{i=t}^{b}$ Force(i)



Assuming v6 assigned to time step 1

Total force = 0.25

Assuming v6 assigned to time step 2

Total force = -1

Better choice – want to reduce force in the minimum resource under latency-constraint

Distribution graph for the multiplier

	0	1	2	3	4
1			2.8		
2			2.3		
3			0.8		
4			0		

Time frame and operation probability for v_6 and v_7

v_6 = [1, 2], width = 2
p(1)=0.5, p(2)=0.5, p(3)=0, p(4)=0

v_7 = [2, 3], width = 2
p(1)=0, p(2)=0.5, p(3)=0.5, p(4)=0

Force-Directed Scheduling (FDS)

- Force-Directed Scheduling
  - Minimum resource under latency constraint

```
FDS( G(V,E), λ ){
  repeat {
    Compute the time frames;
    Compute the operations and type probabilities;
    Compute the self-forces, predecessor/successor forces and total forces;
    Schedule the operation with least force and update its time-frame;
  } until (all operations scheduled);
  return (t);
}
```

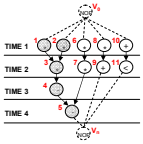
At each iteration time frame, probabilities, and forces need to be recalculated

Forces relate to concurrency – we choose lowest force so we can minimize number of resources

Results have shown FDS superior to list scheduling, but run time are long for larger graph (limited usage)

Force-Directed Scheduling (FDS)

- Previous example only looked at v6
- Algorithm tells us to calculate ALL unscheduled nodes, then schedule operation assignment with smallest force



Conclusion

- Looked at a couple more heuristic scheduling algorithms
  - List_R
  - Force Directed