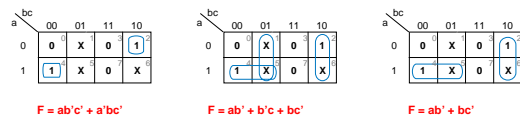


Lecture 8

Quine-McCluskey with Don't Cares, Iterated Consensus, Row/Column Dominance

K-map with Don't Cares

- Consider $F(a, b, c) = \Sigma m(2, 4) + \Sigma d(1, 5, 6)$
- What should we do with the don't cares?
 - Include d.c. if it helps to further minimize the cover (m5, m6)
 - Don't need to include d.c. if it doesn't help (better to exclude m1)



How do we apply these ideas to Quine-McCluskey?

Quine-McCluskey with Don't Cares

Example 1

- $F(a, b, c) = \Sigma m(2, 4) + \Sigma d(1, 5, 6)$

Step 1: Find all the prime implicants

- List all elements of on-set and don't care set, represented as a binary number
- Mark don't cares with "D"

```

G1 (1) 001 D
    (2) 010
    (4) 100
-----
G2 (5) 101 D
    (6) 110 D
    
```

Quine-McCluskey with Don't Cares

Example 1

Step 1: Find all the prime implicants (cont')

- Compare each entry in G_i to each entry in G_{i+1}
 - If they differ by 1 bit, we can apply the uniting theorem and eliminate a literal
 - If both values are don't cares, retain "D", otherwise no need to mark
 - Add check to implicant to remind us that it is not a prime implicant

G_1	(1) 001 D ✓	G_1	(1,5) -01 D	no new implicants are generated – end of step 1
	(2) 010 ✓		(2,6) -10	
	(4) 100 ✓		(4,5) 10-	we have found all prime implicants (ones without check marks)
G_2	(5) 101 D ✓		(4,6) 1-0	
	(6) 110 D ✓			

ECE 474a/575a
Susan Lyshecky

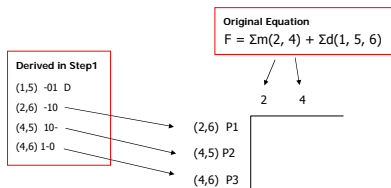
4 of 39

Quine-McCluskey with Don't Cares

Example 1

Step 2: Create Prime Implicant Chart to find all essential prime implicants

- Minterms are added as columns in the table
- Prime implicants **not** marked as "D" are added as rows



ECE 474a/575a
Susan Lyshecky

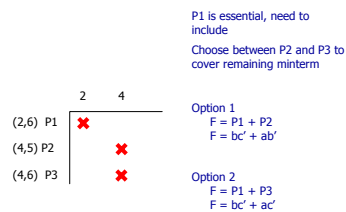
5 of 39

Quine-McCluskey with Don't Cares

Example 1

Step 2: Create Prime Implicant Chart to find all essential prime implicants

- Place "X" in a row if the prime implicant covers the minterm
- Essential prime implicants are found by looking for rows with a single "X"
 - If minterm is covered by one and only one prime implicant – it's an essential prime implicant
- Add essential prime implicants to the cover



ECE 474a/575a
Susan Lyshecky

6 of 39

Iterated Consensus/Complete Sum

- Consider $F(x, y, z) = yz + xy + y'z + xyz + x'z'$

- According to tabular minimization
 - First expanded product term into minterms
 - Then start comparing pairs to determine prime implicants



- Some of the work already done!
 - Instead we can take existing expression and determine the complete sum

Iterated Consensus/Complete Sum

Def: A complete sum is a SOP formula composed of all prime implicants of the function

Thm: A SOP formula is a complete sum if and only if

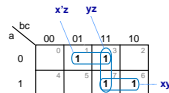
- No term includes any other term
- The consensus of any two terms of the formula either does not exist or is contained in some other term of the formula

Iterated Consensus/Complete Sum

What is consensus?

In Boolean Algebra, consensus is defined as

- $xy + x'z + yz = xy + x'z$
- $(x+y)(x'+z)(y+z) = (x+y)(x'+z)$



K-map shows yz already covered by other two primes

Proof: $xy + x'z + yz = xy + x'z$

$$= xy + x'z + (x + x')yz$$

$$= xy + x'z + xyz + x'yz$$

$$= (xy + xyz) + (x'z + x'yz)$$

$$= xy(1 + z) + x'z(1 + y)$$

$$= xy(1) + x'z(1)$$

$$= xy + x'z$$

Iterated Consensus to Find Complete Sum

- Methodology to convert SOP function to complete sum
 - Start with arbitrary SOP form
 - Add consensus pair of all terms not contained in any other term
 - Compare new terms with existing and among other new terms to see if any new consensus terms can be generated
 - Remove all terms contained in some other term

Repeat 2 – 4 until no change occurs

Iterated Consensus to Find Complete Sum

Example 3

- $F = yz + x'y + y'z' + xyz + x'z'$
 - Start with arbitrary SOP form ✓
 - Add consensus pair of all terms not contained in any other term

$yz + x'y = \text{NO}$	$x'y + y'z' = x'z' \text{ (INCL)}$	$y'z' + xyz = xzz' \rightarrow 0, \text{NO}$
$yz + y'z' = yy' \rightarrow 0, \text{NO}$	$x'y + xyz = yz \text{ (INCL)}$	$y'z' + x'z' = \text{NO}$
$yz + xyz = \text{NO}$	$x'y + x'z' = \text{NO}$	
$yz + x'z' = x'y \text{ (INCL)}$		$xyz + x'z' = xx'y \rightarrow 0, \text{NO}$
 - Compare new terms with existing and among other new terms to see if any new consensus terms can be generated

No new terms generated
 - Remove all terms contained in some other term

$$yz + x'y + y'z' + xyz + x'z'$$

$$yz + x'y + y'z' + x'z'$$

since there is a change you will need to start again – you will find in the next iteration no change occurs

Iterative vs. Recursive

- Iterative approach
 - Repetitive procedure used to add new consensus terms
- Recursive approach
 - Also repetitive, but we are trying to keep simplifying problem until solution is easy

Iterated Consensus Methodology

- Start with arbitrary SOP form
 - Add consensus pair of all terms not contained in any other term
 - Compare new terms with existing and among other new terms to see if any new consensus terms can be generated
 - Remove all terms contained in some other term
- Repeat 2 – 4 until no change occurs

Quine-McCluskey Overview

Quine-McCluskey Algorithm

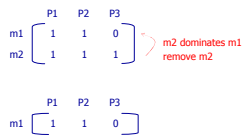
- | | | |
|---|---|---|
| (1) Find all prime implicants | How is each step currently done?
Tabular Minimization | Are there alternatives?
Iterated Consensus to find complete sum |
| (2) Find all essential prime implicants | Prime Implicant Chart (row with single 'X') | Constraint Matrix (basically same thing except axis switched) |
| (3) Select a minimal set of remaining prime implicants that covers the on-set of the function | Petrick's Method | Row/Column Dominance |

ECE 474a/575a
Susan Lyshecky

31 of 39

Row Dominance (Constraint)

- If a row r_i in a constraint matrix has all the ones of another row r_j , we say r_i **dominates** r_j
- r_i is unneeded and all **dominating** row can be removed
 - Absorption property $x \cdot (x + y) = x$

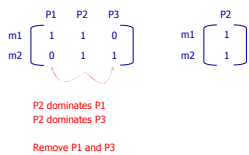


ECE 474a/575a
Susan Lyshecky

32 of 39

Column Dominance (Variable)

- If column P_i has all the ones of another column P_j , and the cost of P_i is not greater than P_j , we say P_i **dominates** P_j
- The **dominated** column can be removed



- Assumes P2 does not cost more than P1 or P3
- What is the cost of a column?
 - Each prime implicant (col) corresponds to one AND gate
- Our Choices**
 - We could say each column = 1 gate and everyone's the same.
 - We could include number of literal, then a prime with 5 literals cost more than 3

ECE 474a/575a
Susan Lyshecky

33 of 39

Reduction Techniques Using Row/Col Dominance

1. Remove rows covered by "essential columns" (i.e. essential prime implicants)
2. Remove rows through row dominance (dominating row removed)
3. Remove columns through column dominance (dominated column removed)

Re-iterate 1-3 until no further simplification is possible

ECE 474a/575a
Susan Lysdecky

34 of 39

Reduction Techniques Using Row/Col Dominance

Example 6

	P1	P2	P3	P4	P5	P6
m1	1	1				1
m2	1	1				
m3			1	1		
m4		1	1	1		
m5					1	1
m6			1	1	1	1

(1A) No essential columns to remove

	P1	P2	P3	P4	P5	P6
m2	1	1				
m3			1	1		
m5					1	1

(2A) Row dominance

m1 dominates m2

m4 dominates m3

m6 dominates m5

Remove the **dominating** rows

ECE 474a/575a
Susan Lysdecky

35 of 39

Reduction Techniques Using Row/Col Dominance

Example 6

	P1	P2	P3	P4	P5	P6
m2	1	1				
m3			1	1		
m5					1	1

Note: P5 tell us nothing, you can remove to simplify if you want

(3A) Column dominance

P2 dominates P1

P2 dominates P3

P4 dominates P5, vice versa

Remove the **dominated** cols

	P2	P4
m2	1	
m3		1
m5		1

(1A) Essential Columns

ECE 474a/575a
Susan Lysdecky

36 of 39

Reduction Techniques Using Row/Col Dominance

Example 6

$$\begin{matrix} & \begin{matrix} P2 & P4 \end{matrix} \\ \begin{matrix} m2 \\ m3 \\ m5 \end{matrix} & \begin{bmatrix} 1 & \\ 1 & \\ & 1 \end{bmatrix} \end{matrix}$$

(1A) Essential Columns

P4 only column to cover m5
essential prime implicant = {P4}

P2 only column to cover m2, m3
essential prime implicant = {P4, P2}

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

Simplify matrix

Matrix empty – no further simplification possible

$$\text{Cover} = P2 + P4$$

ECE 474a/575a
Susan Lysecky

37 of 39

Reduction Techniques Using Row/Col Dominance

What happens when matrix cannot be simplified?

- No rows left
 - We have a terminal case and solved the problem
- Rows left
 - Problem is cyclic
 - Alternative techniques such as divide-and-conquer or branch-and-bound are needed
 - (Or guess, or use Petrick's)

ECE 474a/575a
Susan Lysecky

38 of 39

Conclusion

- Quine-McCluskey with Don't Cares
- Alternative methods to perform Quine-McCluskey algorithm
- Iterated consensus (iterative and recursive)
- Generate a complete sum
- Row/Column Dominance

ECE 474a/575a
Susan Lysecky

39 of 39
