Shannon Expansion Examples

$$F(x_1, x_2, \ldots, x_n) = x_1 \cdot F(1, x_2, \ldots, x_n) + x_1' F(0, x_2, \ldots, x_n)$$

① Expand $F(a,b,c) = abc + a'b'c + b'c'$ with respect to variable a

$$F(a,b,c) = a \cdot F(1,b,c) + a' \cdot F(0,b,c)$$

$$\begin{aligned} F(1,b,c) &= (1)bc + (0)b'c + b'c' \\ &= bc + b'c' \end{aligned}$$

// $x \cdot 0 = 0$; $x + 0 = x$

$$\begin{aligned} F(0,b,c) &= (0)bc + (1)b'c + b'c' \\ &= b'c + b'c' \end{aligned}$$

$$\boxed{F(a,b,c) = a(bc + b'c') + a'(b'c + b'c')}$$

② Find the cofactor of $F(a,b,c) = a'b' + ac + b'c$ with respect to $a'c$

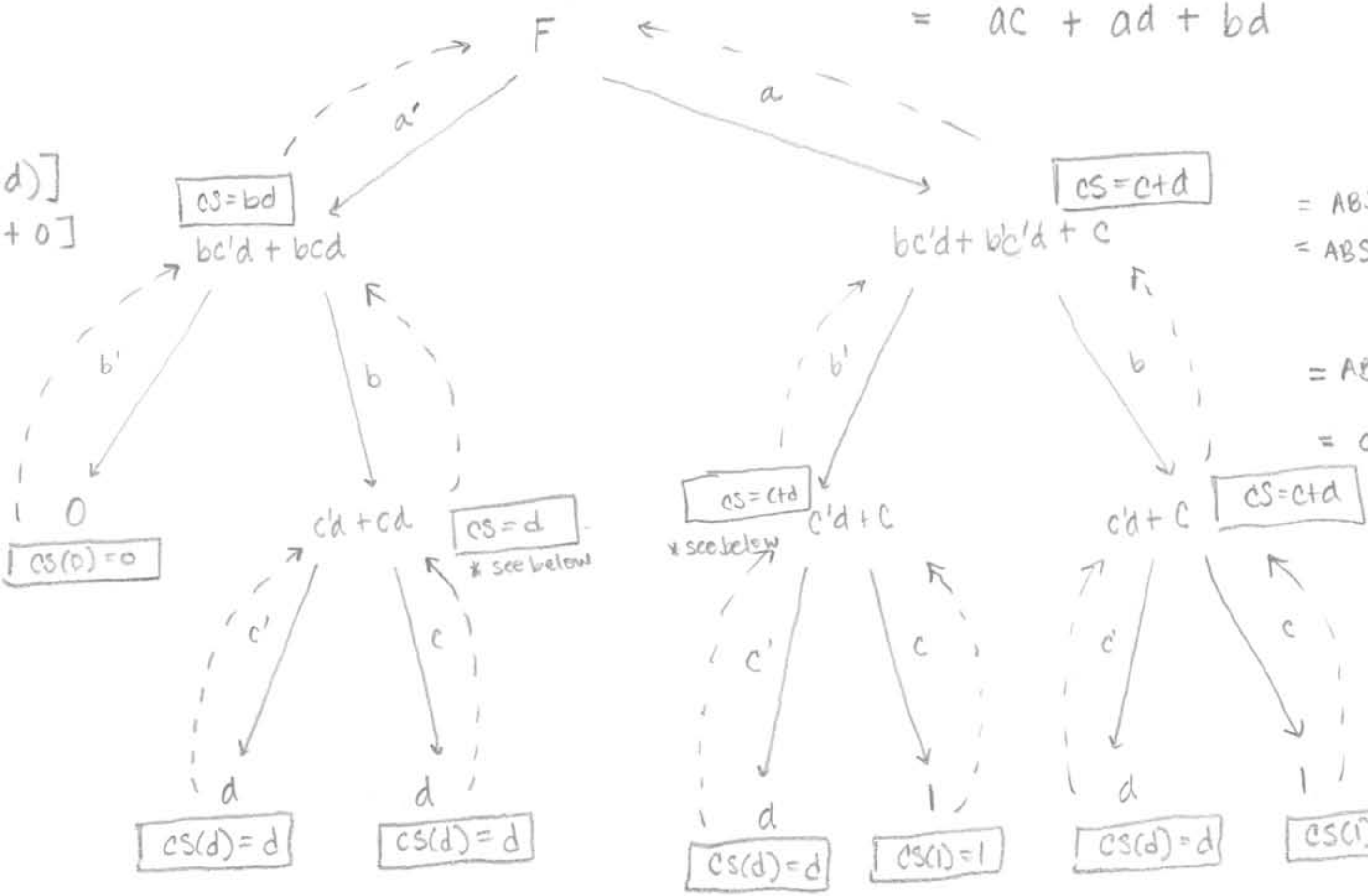
$$\begin{aligned} F(0,b,1) &= (1)b' + (0)(1) + b'(1) \\ &= b' + b' \\ &= b' \end{aligned}$$

$$\boxed{F_{a'c} = b'}$$

Recursive Consensus Example

$F(a,b,c,d) = bc'd + a'bcd + ab'c'd + ac$

$= ABS[(a + bd)(a' + c + d)]$
$= ABS[aa' + ac + ad + a'bd + bcd + bdd]$
$= ABS[ac + ad + a'bd + bcd + bd]$
$= ac + ad + bd$



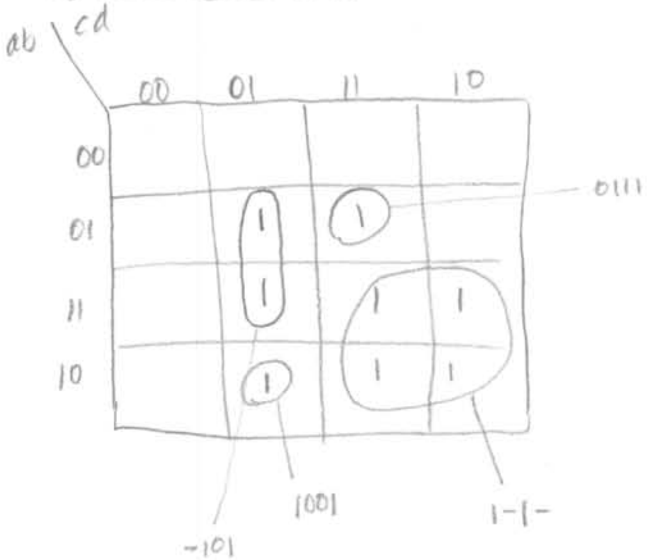
$= ABS[(b+0)(b'+d)]$
$= ABS[bb' + bd + 0 + 0]$
$= ABS[bd]$
$= bd$

$= ABS[(b+c+d)(b'+c+d)]$
$= ABS[bb' + bc + bd + b'c + cc + cd + b'd + cd + dd]$
$= ABS[bc + bd + b'c + c + cd + b'd + d]$
$= c + d$

$= ABS[(x + F_{x'}) \bullet (x' \bullet F_x)]$
$= ABS[(c+d)(c'+d)]$
$= ABS[cc' + cd + c'd + dd]$
$= ABS[cd + c'd + d]$
$= d$

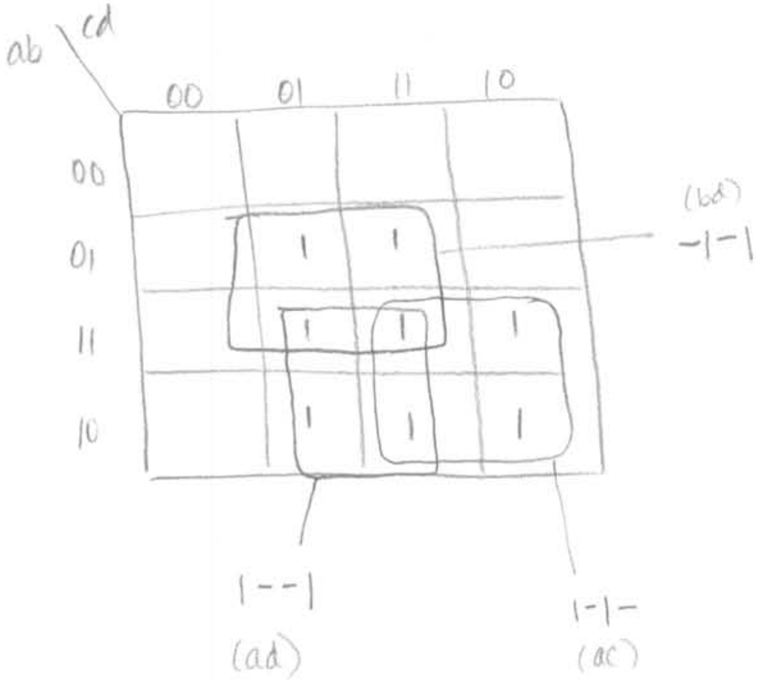
$= ABS[(c+d)(c'+1)]$
$= ABS[cc' + c + c'd + d]$
$= ABS[c + c'd + d]$
$= c + d$

original K-map



bc'd → -101
a'bcd → 0111
ab'c'd → 1001
ac → 1-1-

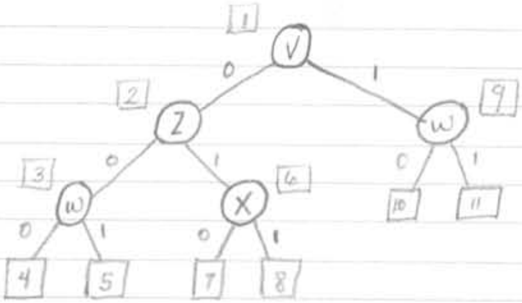
after recursive consensus



ac → 1-1-
ad → 1--1
bd → -1-1

Hatchel Example mentioned in slide #21

$F(v,w,x,y,z) = v'xyz + v'w'x + v'x'z' + v'wxz + w'yz' + vw'z + vwx'z$



* letter in nodes represent variable we expand to 0/1

(1) $F(v,w,x,y,z) = v'xyz + v'w'x + v'x'z' + v'wxz + w'yz' + vw'z + vwx'z$

(2) $F(0,w,x,y,z) = xyz + w'x + x'z' + wxz + w'yz'$

(3) $F(0,w,x,y,0) = w'x + x' + w'y$

(4) $F(0,0,x,y,0) = x + x' + y = \boxed{1}$

(5) $F(0,1,x,y,0) = \boxed{x'}$

(3) $CS(F(0,w,x,y,0)) = ABS((w+1)(w'+x')) = \boxed{w'+x'}$

(6) $F(0,w,x,y,1) = xy + w'x + wx$

(7) $F(0,w,0,y,1) = \boxed{0}$

(8) $F(0,w,1,y,1) = y + w' + w = \boxed{1}$

(6) $CS(F(0,w,x,y,z)) = ABS(z+w'+x')(z'+x)) = \boxed{w'x + w'z' + x'z' + xz}$

(9) $F(1,w,x,y,z) = w'yz' + w'z + wx'z$

(10) $F(1,0,x,y,z) = yz' + z = \boxed{y+z}$

(11) $F(1,1,x,y,z) = \boxed{x'z}$

(9) $CS(F(1,w,x,y,z)) = ABS((w+y+z)(w'+x'z)) = \boxed{w'y + w'z + x'z}$

(1) $CS(F(v,w,x,y,z)) = ABS((v + w'x + w'z' + x'z' + xz)(v' + w'y + w'z + x'z))$

$= vw'y + vw'z + vx'z + v'w'x + w'xy + w'xz + v'w'z' + w'yz' + v'x'z' + v'xz$