

# In-class Exercise 11

(PAR 15)

- Solve for ITE (abcd, 1, c'd + a'd') using the variable ordering  $a \leq b \leq c \leq d$

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ITE(F, G, H){
  (result, terminal_case) = TERMINAL_CASE(F, G, H)

  if (terminal_case) return (result)
  (result, in_computed_table) = COMPUTED_TABLE_HAS_ENTRY(F, G, H)

  if (in_computed_table) return (result)

  v = TOP_VARIABLE(F, G, H)
  T = ITE(Fv, Gv, Hv)
  E = ITE(Fv', Gv', Hv')
  R = FIND_OR_ADD_UNIQUE_TABLE(v, T, E)
  INSERT_COMPUTED_TABLE((F, G, H), R)
  return (R)
}
```

Table	Name	Expression	Equivalent Form
0000	0	0	0
0001	AND(F, G)	FG	ITE(F, G, 0)
0010	F > G	FG'	ITE(F, G', 0)
0011	F	F	F
0100	F < G	F'G	ITE(F, 0, G)
0101	G	G	G
0110	XOR(F, G)	F ⊕ G	ITE(F, G', G)
0111	OR(F, G)	F + G	ITE(F, 1, G)
1000	NOR(F, G)	(F + G)'	ITE(F, 0, G')
1001	XNOR(F, G)	(F ⊕ G)'	ITE(F, G, G')
1010	NOT(G)	G'	ITE(G, 0, 1)
1011	F ≥ G	F + G'	ITE(F, 1, G')
1100	NOT(F)	F'	ITE(F, 0, 1)
1101	F ≤ G	F' + G	ITE(F, G, 1)
1110	NAND(F, G)	(FG)'	ITE(F, G', 1)
1111	1	1	1

$$\begin{aligned} \text{ITE}(F, G, H) &= FG + F'H \\ &= \text{ITE}(v \text{ ITE}(F_v, G_v, H_v), \text{ITE}(F_v', G_v', H_v')) \end{aligned}$$

Terminal Cases

$$\text{ITE}(1, F, G) = F \quad \text{ITE}(F, 1, 0) = F$$

$$\text{ITE}(0, G, F) = F \quad \text{ITE}(G, F, F) = F$$

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- Solve for ITE (abcd, 1, c'd + a'd') using the variable ordering  $a \leq b \leq c \leq d$

$$= \text{ITE}(\text{abcd}, 1, c'd + a'd')$$

$$= \text{ITE}(a, \text{ITE}(\text{abcd}_a, 1_a, c'd + a'd'_a), \text{ITE}(\text{abcd}_a, 1_a, c'd + a'd'_a))$$

$$= \text{ITE}(a, \text{ITE}(\text{bcd}, 1, c'd), \text{ITE}(0, 1, c'd + d')) \quad // \text{Simplify } \text{ITE}(0, 1, c'd + d'_a) = c'd + d'$$

$$= \text{ITE}(a, \text{ITE}(\text{bcd}, 1, c'd), c'd + d') \quad // \text{Exapnd w.r.t b}$$

$$= \text{ITE}(a, \text{ITE}(b, \text{ITE}(\text{bcd}_b, 1_b, c'd_b), \text{ITE}(\text{bcd}_b, 1_b, c'd_b)), c'd + d')$$

$$= \text{ITE}(a, \text{ITE}(b, \text{ITE}(cd, 1, c'd), \text{ITE}(0, 1, c'd)), c'd + d') \quad // \text{Simplify } \text{ITE}(0, 1, c'd) = c'd$$

$$= \text{ITE}(a, \text{ITE}(b, \text{ITE}(cd, 1, c'd), c'd), c'd + d')$$

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- Continued

$$\begin{aligned} &= \text{ITE}(a, \text{ITE}(b, \underline{\text{ITE}(cd, 1, c'd)}, c'd), c'd + d') && // \text{Exapnd w.r.t } c \\ &= \text{ITE}(a, \text{ITE}(b, \text{ITE}(c, \text{ITE}(cd_c, 1_c, c'd_c), \text{ITE}(cd_{c'}, 1_{c'}, c'd_{c'}), c'd), c'd + d') \\ &= \text{ITE}(a, \text{ITE}(b, \text{ITE}(c, \underline{\text{ITE}(d, 1, 0)}, \text{ITE}(0, 1, d), c'd), c'd + d') && // \text{Simplify } \text{ITE}(d, 1, 0) = d \\ &= \text{ITE}(a, \text{ITE}(b, \text{ITE}(c, d, \underline{\text{ITE}(0, 1, d)}, c'd), c'd + d') && // \text{Simplify } \text{ITE}(0, 1, d) = d \\ &= \text{ITE}(a, \text{ITE}(b, \underline{\text{ITE}(c, d, d)}, c'd), c'd + d') && // \text{Simplify } \text{ITE}(c, d, d) = d \\ &= \text{ITE}(a, \text{ITE}(b, d, \underline{c'd}), c'd + d') && // \text{Put } c'd \text{ into ITE format } (FG = \text{ITE}(F, G, 0)) \\ &= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c', d, 0), c'd + d') && // \text{Exapand } \text{ITE}(c', d, 0) \text{ w.r.t } c \\ &= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, \text{ITE}(c'_c, d_c, 0_c), \text{ITE}(c'_{c'}, d_{c'}, 0_{c'})), c'd + d') \\ &= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, \underline{\text{ITE}(0, d, 0)}, \underline{\text{ITE}(1, d, 0)}), c'd + d') && // \text{Simplify} \\ &= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, 0, d), c'd + d') \end{aligned}$$

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- Continued

$$= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, 0, d), \underline{c'd + d'})) \quad // \text{ Put into ITE format } (F + G = \text{ITE}(F, 1, G))$$

$$= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, 0, d), \underline{\text{ITE}(c'd, 1, d')})) \quad // \text{ Expand } \text{ITE}(c'd, 1, d') \text{ w.r.t } c$$

$$= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, 0, d), \text{ITE}(c, \text{ITE}(c'd_c, 1_c, d'_c), \text{ITE}(c'd_c, 1_c, d'_c))))$$

$$= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, 0, d), \text{ITE}(c, \underline{\text{ITE}(0, 1, d')}, \text{ITE}(d, 1, d')))) \quad // \text{ Simplify } \text{ITE}(0, 1, d') = d'$$

$$= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, 0, d), \text{ITE}(c, d', \underline{\text{ITE}(d, 1, d')}))) \quad // \text{ Simplify } \text{ITE}(d, 1, d') = 1$$

If  $d=1$ , then output = 1

If  $d=0$ , then output =  $d'$  ( $0' = 1$ )

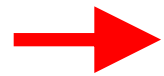
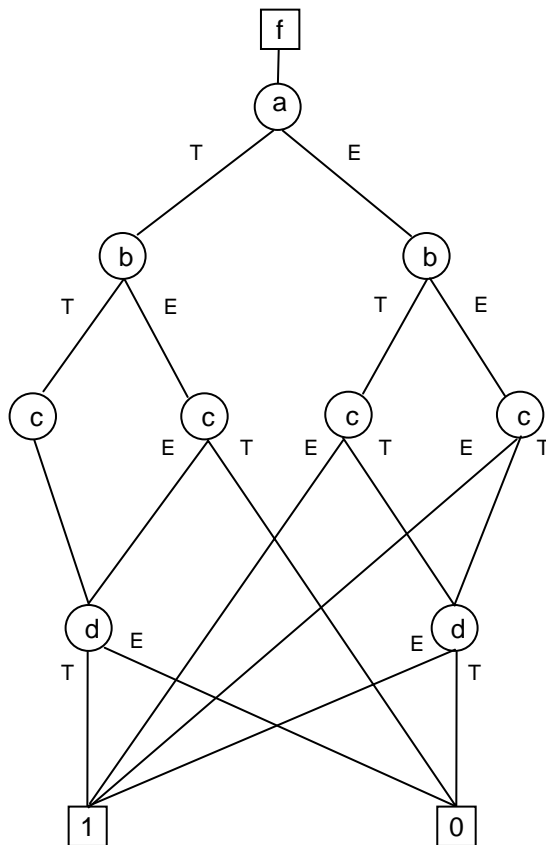
$$= \text{ITE}(a, \text{ITE}(b, d, \text{ITE}(c, 0, d), \text{ITE}(c, d', 1)))$$

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- Solution from ICE10/PAR14

ITE gives us the reduced solution to start with  
= ITE (a, ITE(b, d, ITE(c, 0, d), ITE(c, d', 1)))



After manual  
simplification

