Importance Sampling for Loop-Free Decoding Trees with Application to Low-Density Parity-Check Codes[1]

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Abstract — **We propose an importance sampling scheme for the decoding of loop-free decoding trees. It is shown that this scheme is asymptotically efficient and that, for an arbitrary tree and a given estimation precision, the number of simulation runs needed is inversely proportional to the standard deviation of the channel noise. This work has its application to the decoding of low-density parity-check codes.**

I. Single Parity-Check Codes

The principle of importance sampling (IS) with mean translation is to bias the noise toward the decoding error boundary so that the decoder errs more frequently, and unbiased estimation can be achieved by appropriate weighting in the estimator expression.

For a single parity-check code (SPCC) of length n under message-passing decoding, it can be graphically illustrated (for a length-3 code) that the error boundary ∂E of a single bit, say v_1, is dominated by $n-1$ sub-boundaries $\partial E^{(k)}$ corresponding to the other $n-1$ code bits. This is due to the fact that, with the message-passing algorithm, the message passed to v_1 is dominated by the message with the least magnitude (the *min* part in the min-sum approximation). Hence, each sub-boundary $\partial E^{(k)}$ can be approximated by a flat plane $m_1^i + m_k^i = 0$, where m_k^i is the *a priori* message of v_k. A single bias $\bar{b}^{(k)} = (1, 0, \ldots, 0, \overset{k\text{th}}{1}, 0, \ldots, 0)$ can be determined for this plane. This resembles the asymptotically efficient IS bias for binary decisions in [1], for which the required number of simulated bits to achieve a relative precision ϵ can be shown to be $L \simeq \sqrt{\frac{\pi}{2}}\frac{1}{\epsilon^2\sigma} \simeq \frac{1.25}{\epsilon^2\sigma} = \frac{C_L}{\epsilon^2\sigma}$.

Tab. 1 gives IS simulation results for a length-3 SPCC. The agreement between the IS and Monte Carlo (MC) estimation results is (and can only be) observed at 10 dB. The last column indicates that this scheme is asymptotically efficient, with a slightly larger constant $C_L = 1.6$ (empirically determined) as compared with $C_L = 1.25$ for binary decisions.

Tab. 1: IS simulation results for SPCC ($n = 3$). $\epsilon = 10\%$.

E_b/N_0	$\hat{P}_b$ (MC)	$\hat{P}_b$ (IS)	L	C_L
10 dB	2.4×10^{-7}	2.4×10^{-7}	588	1.6
20 dB	n/a	6×10^{-60}	1840	1.6

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II. Decoding Trees

For simplicity of explanation, a regular decoding tree is treated in Fig. 1, where circles are variable nodes and squares are check nodes. The message-passing starts from the bottom of the tree until it reaches the root node v_1.

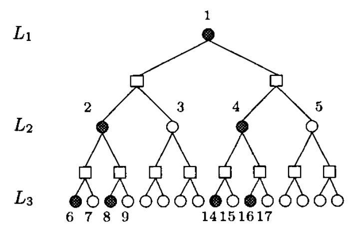

Fig. 1: A biased tree.

With the min-sum approximation, the error boundary ∂E for the decoding of v_1 is dominated by four (2×2) sub-boundaries, one of which, $\partial E^{(2,4)}$, is characterized by $|m_2^o| \le m_3^o$ and $|m_4^o| \le m_5^o$, where m_k^o is the *a posteriori* message of v_k. The sub-boundary $\partial E^{(2,4)}$ can be further broken down to form $\partial E^{(2,4,6,8,14,16)}$ which can be approximated by

$$m_1^i + m_2^i + m_4^i + m_6^i + m_8^i + m_{14}^i + m_{16}^i = 0,$$

and a '1' is added to each node to form the bias toward this sub-boundary, as indicated in Fig. 1 by shaded nodes.

Simulation results for Fig. 1 are listed in Tab. 2. The last column shows a decreasing C_L with the signal-to-noise ratio (SNR). This is because in the low-SNR region, the flat-boundary assumption is far from being accurate, resulting in an unpredictable and high C_L value.

Tab. 2: IS simulation results for decoding tree. $\epsilon = 10\%$.

E_b/N_0	$\hat{P}_b$ (MC)	$\hat{P}_b$ (IS)	L	C_L
3 dB	8.6×10^{-5}	8.7×10^{-5}	10^5	400
4 dB	7.3×10^{-7}	6.7×10^{-7}	27000	135
8 dB	n/a	1.6×10^{-25}	2300	7.4
12 dB	n/a	2.9×10^{-75}	2400	4.8

References

[1] D. Lu and K. Yao, "Improved importance sampling technique for efficient simulation of digital communication systems," *IEEE J. Select. Areas Commun.*, vol. 6, pp. 67–75, Jan. 1988.

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