TIME DELAY ESTIMATION FOR SPREAD SPECTRUM SIGNALS USING GENERALIZED CROSS CORRELATION

William E. Ryan

American Satellite Company
1801 Research Blvd.
Rockville, MD 20850

J.M. Schumpert

The Analytic Sciences Corp.
8301 Greensboro Drive, Suite 1100
McLean, VA 22102

ABSTRACT

The generalized cross correlation approach to time delay estimation is tested with direct sequence and frequency hopping spread spectrum signals in white and narrowband noise. The performance of six correlation filters [1] are evaluted via Monte Carlo simulation where sample variances and mean-square errors (MSE) are estimated for several signal-to-noise ratios. The results allow one to choose an optimal filter according to a minimum mean square error criterion. Additionally, the sample variances are compared to the lower bound developed in [2].

INTRODUCTION

A singal $s(t)$ received at two spatially seperated sensors may be modeled by

$$r_1(t) = s(t) + n_1(t) \qquad (1a)$$

$$r_2(t) = A\, s(t-D) + n_2(t) \qquad (1b)$$

where A is the relative amplitude of the signal at receiver 2, D is the relative delay, and $n_1(t)$ and $n_2(t)$ are the contaminating noises. It is assumed that the observation time T is sufficiently short to ensure that A and D are constant and that the model is stationary.

An estimate $\hat{D}$ of the delay may be obtained by forming the cross correlation of $r_1(t)$ and $r_2(t)$ and choosing $\hat{D}$ as the time lag corresponding to the correlation peak. Because of the finite observation time, the cross correlation may only be estimated and under an ergodicity assumption an estimate is the time average

$$R_{r_1r_2}(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} r_1(t)\, r_2(t+\tau)dt \qquad (2)$$

The estimate of D may be enhanced if $r_1(t)$ and $r_2(t)$ are each filtered before time averaging. It is easily shown that the filtered results $x(t) = r_1(t)*h_1(t)$ and $y(t) = r_2(t)*h_2(t)$ have a correlation function estimate given by

$$\hat{R}_{xy}(t) = \hat{R}_{r_1r_2}(t)*h_1(-t)*h_2(t) \qquad (3)$$

where * denotes convolution.

The function $\hat{R}_{xy}(t)$ is an estimate of what is known as the generalized correlation function. To exploit digital signal processing algorithms which use the FFT, $\hat{R}_{xy}(t)$, is usually computed in the frequency domain. A frequency domain version of (3) is

$$\hat{G}_{xy}(f) = \hat{G}_{r_1r_2}(f)\, H_1^*(f)\, H_2(f) \qquad (4)$$

where here * as a superscript indicates the complex conjugate and the transorm pairs in (3) and (4) are obvious.

Several filters or window functions $W(f) = H_1^*(f)\, H_2(f)$ have been proposed in the literature and the ones used in this study are contained in [1] and listed in Table I. The purpose of this research is to evaluate the effectiveness of the generalized cross correlator (GCC), when $s(t)$ is a spread spectrum signal, when the windows used are those of Table I, and when the noise processes are either correlated or uncorrelated.

APPLICATION TO SPREAD SPECTRUM SIGNALS

A direct sequence (DS) spread spectrum (SS) signal can be expressed [3] as

$$s(t) = d(t)\, PN(t) \cos(\omega_o t) \qquad (5)$$

where $d(t)$ is binary data (± 1), $PN(t)$ is a pseudo-noise sequence (± 1) and ω_o is the carrier frequency. The phase transitions in $PN(t)$ are typically orders of magnitude faster than those in $d(t)$ and produce the spreading effect.

In frequency hopping (FH) SS modulation $PN(t)$ is used to pseudorandomly hop a binary frequency-shift keyed (BFSK) signal over a band of frequencies. An FH SS signal can be expressed [3] as

$$S(t) = \cos[(\omega_o + \omega_h(t) + \omega_d\, d(t))t] \qquad (6)$$

where $2\omega_d$ is the separation between the two FSK tones, the hop frequency $\omega_h(t)$ is a function of $PN(t)$ and ω_o and $d(t)$ are as above.

Since bandpass signals are to be processed, the GCC must be implemented to accomodate bandpass signals. Typically, in estimating the cross correlation of two bandpass processes. the cross correlation estimate of their complex envelopes is used. In [4], it is shown that an estimate of the actual bandpass correlation function may be obtained by multiplying the complex envelope correlation function estimate by $\exp(j\omega_o t)$ and taking the real part of the result. In view of this technique, a bandpass version of the GCC can be realized as illustrated in Fig. 1.

A lower bound for the variance of the estimate $\hat{D}$ derived from (2) (i.e. no filtering) was obtained by Ianniello [2]. The bound was verified by simulation in [5] for the case where s(t) is a low pass filtered Gaussian noise process and $n_1(t)$ and $n_2(t)$ are independent Gaussian white noise process. The bound for the variance of D is a combination of the Cramer Rao (CR) bound, which is a lower bound on the variance of any unbiased estimate, and a term which is essentially the variance of the estimate under a uniform probability density condition. The two terms of this combination are weighted by factors which depend on the probability of anomaly as follows:

$$\sigma_I^2 = \frac{1}{3} PT_0^2 + (1-P)\sigma_{CR}^2 \qquad (7)$$

where T_0 is half the width of the correlation window, σ_{CR}^2 is the appropriate Cramer-Rao bound, and the probability of anomaly is approximated as

$$P=1 - \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} \exp\left[-(x-\alpha)^2/2\right]\left\{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta} \exp(-y^2/2)dy\right\}^{M-1} dx$$

and

$$\alpha = \frac{\sqrt{1.5BT}\ \mathrm{SNR}}{\sqrt{\mathrm{SNR}^2 + (1+\mathrm{SNR})^2}} \qquad (8)$$

$$\beta = \frac{\sqrt{2\mathrm{SNR}(\mathrm{SNR} + 1) + 1}}{(\mathrm{SNR} + 1)}$$

$$M = BT_0$$

Expressions for the CR bounds for DS and FH spread spectrum signals were derived in [6] and are

$$\sigma_{DS}^2 \geq \left[2T_0 \int_0^{\infty} (2\pi f)^2 \frac{\mathrm{SNR}^2\ \mathrm{sinc}^4[T_c(f-f_0)]}{2\ \mathrm{SNR}\ \mathrm{sinc}^2[T_c(f-f_0)]+1} df\right]^{-1} \qquad (9a)$$

$$\sigma_{FH}^2 \geq \left(\frac{3}{8\pi^2 T}\right)\left(\frac{1+2\ \mathrm{SNR}}{\mathrm{SNR}^2}\right)\left(\frac{1}{f_2^3 - f_1^3}\right) \qquad (9b)$$

where T is the observation time, T_c is the chip rate, f_0 is the carrier frequency, and the spectrum extends from f_1 to f_2. The bound is illustrated for DS signals in the next section where we discuss the results of the simulation of the GCC in Fig. 1.

THE SIMULATION AND RESULTS

The estimates of the auto and cross spectra of the complex envelopes of $r_1(t)$ and $r_2(t)$ were produced by the method of averaging modified periodograms. From these estimates, an estimate of the generalized cross correlation was obtained in the manner depicted in Fig. 1. Blocks of length 1024 were processed, 512 of these samples being appended zeros to reduce the effect of aliasing due to circular correlation.

An example (Roth) correlogram is shown in Fig. 2 where s(t) is DS in white Gaussian noise (WGN), the signal-to-noise ratio (SNR) is -10dB, the (null-to-null) bandwidth B is 200 MHz, the observation time T is 15μs, and the delay D is 128 samples. A similar correlogram results for the FH case only when the SNR is greater than about-2dB. The reason for the large disparity in performance is that for practical observation times (at least near real-time processing is desired) the FSK signal is only hopped over a fraction of the toatal number of slots possible, thus reducing the BT product significantly.

Sample first-and-second-order moments were computed from the simulation results for various SNR's. It was found that the estimates produced were biased. The biases were removed in order to compare the variances to those predicted by the Ianniello bound for unbiased estimates. For DS signals in WGN the results for the Basic "window" agreed best with the bound and MSE's and variances are plotted against the bound in Fig. 3 for several SNR's. For the other windows, the variances were typically 10dB above the bound and the MSE's about 10dB below the bound. The FH results were not compared to the bound because of the aforementioned defficulty with the dependency of bandwidth of observation time.

Some GCC windows have the ability to null out narrowband interference. An example magnitude response for the SCOT window for DS signalling in white and narrowband noise is given in Fig. 4. Similar responses are obtained for the Roth, PHAT and ML windows for both DS and FH. The Basic window is obviously flat. The Eckart window produced peaks rather than nulls because an approximation [1] was used since the form in Table I requires prior knowledge of the signal and noise spectra.

SUMMARY

For DS signals we found that the Eckart and Basic correlator yield the lowest MSE's in white noise where as the SCOT, PHAT and Roth correlators perform the best when narrowband noise is added to the white noise. For FH signals the Basic and ML correlators perform the best in white noise while the SCOT and Roth perform the best in a combination of white and narrowband noise.

REFERENCES

1. C.H. Knapp and G.C. Carter, "The Generalized Correlation Method for Estimation of Time Delay," IEEE Trans. ASSP, vol. ASSP-24, pp. 320-327, August 1976.

2. J.P. Ianniello, "Time Delay Estimation Via Cross Correlation in the Presence of Large Estimation Errors," IEEE Trans. ASSP, vol. ASSP-30, pp. 998-1003, December 1982.

3. Special Issue on Spread Spectrum Communications, IEEE Trans. Comm., vol. Com-25, August 1977.

4. G.W. Johnson, D.E. Olms, and M.L. Hampton, "Broadband Correlation Processing," Proc. 1983 Int. Conf. ASSP, pp. 583-586.

5. K. Scarbrough, R. Tremblay and G. Carter, "Performance Predictions for Coherent and Incoherent Processing Techniques of Time Delay Estimation," IEEE Trans. ASSP, vol. ASSP-31, pp. 1191-1196, October 1983.

6. W.E. Ryan, "Generalized Cross Correlation Performance with Spread Spectrum Signals," M.S. Thesis, Department of Electrical Engineering, University of Virginia, Charlottesville, [August 1984].

Table 1 List of the correlators used in the simulation. $\gamma_{12}(f)=G_{r_1r_2}(f)/[G_{r_1r_1}(f)G_{r_2r_2}(f)]^{\frac{1}{2}}$

Name	$W(f)=H_1^*(f)H_2(f)$
Basic	1.0
Roth	$1/G_{r_1r_1}(f)$
SCOT	$1/\sqrt{G_{r_1r_1}(f)G_{r_2r_2}(f)}$
PHAT	$1/\lvert G_{r_1r_2}(f)\rvert$
Eckart	$G_{ss}(f)/G_{n_1n_1}(f)G_{n_2n_2}(f)$
ML	$\lvert\gamma_{12}(f)\rvert^2/\lvert G_{r_1r_2}(f)\rvert\{1-\lvert\gamma_{12}(f)\rvert^2\}$

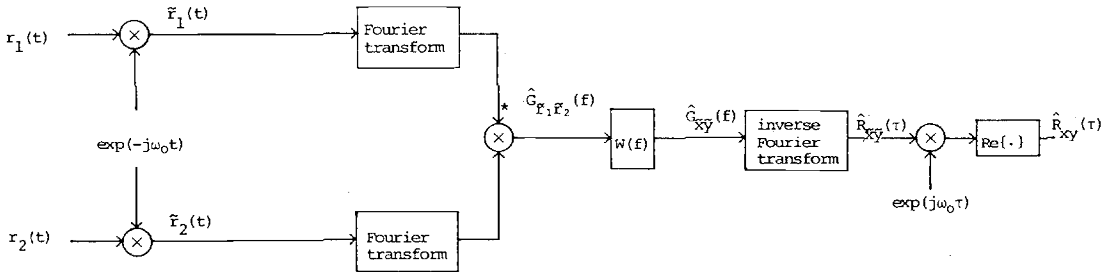

Figure 1. Bandpass version of the GCC.

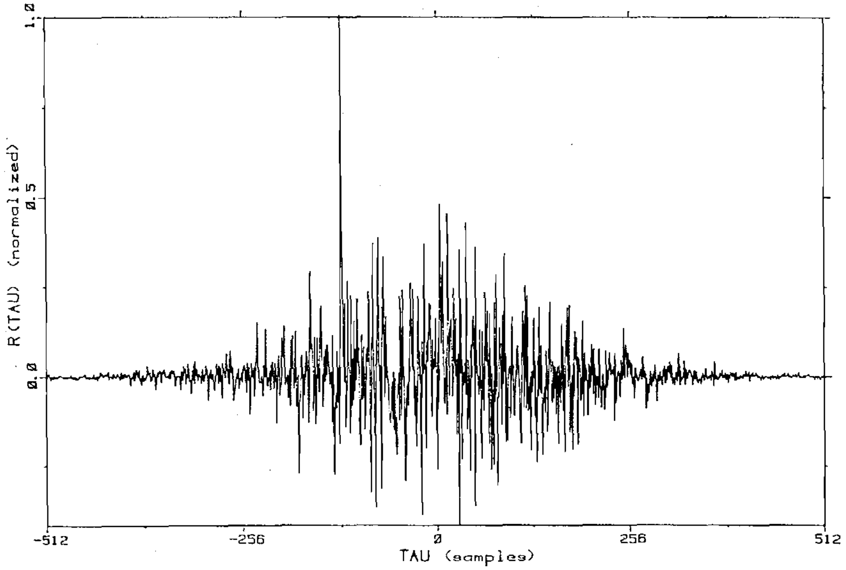

Figure 2. Example correlogram; Roth window, DS signalling, white noise.

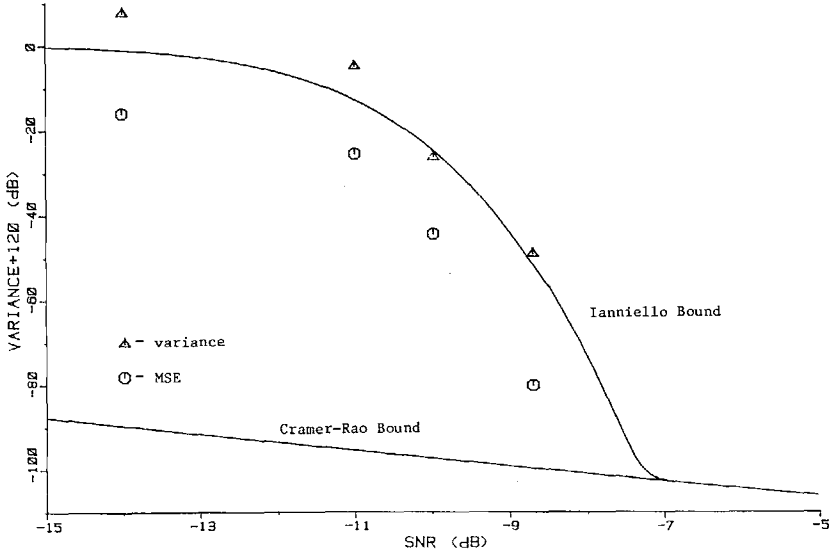

Figure 3. Experimental results versus theoretical bounds.

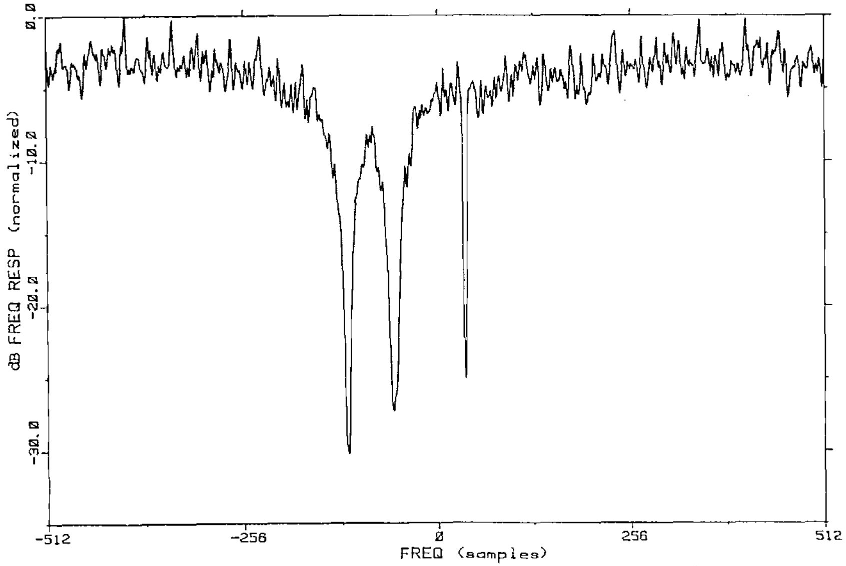

Figure 4. SCOT window magnitude; DS signalling in white and narrowband noise (three narrowband noise processes).