Finite Shift-Invariant Optical Orthogonal Codes for Quasi-Synchronous Communication Systems

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***Abstract*— This paper considers a distribution/aggregation network that employs optical code division multiple access (OCDMA) to transmit information to a head-end node in a metropolitan area network. The head-end node transmits synchronization pulses and the nodes employ ranging techniques to offset the propagation delay, resulting in a quasi-synchronous reception at the receiver. The paper develops a methodology to design orthogonal OCDMA codes for such quasi-synchronous communication systems. The paper also describes the receiver design and computes the bit error rate of such systems.**

I. Introduction

Tremendous progress in optical transmission and networking technology has resulted in a glut of bandwidth in the backbone wide-area networks. The last-mile pathway, however, is still extremely narrow. Cable- and DSL-based Internet access only provide bandwidths of the order of a few hundred Kb/s to a few Mb/s, and even these technologies have been slow to catch on in the U.S. According to [1], only about 10% of U.S. households have broadband access as compared to over 52% of households with Internet access.

Concurrent with the capacity expansion in the wide-area networks, there has been an increase in desktop data generation rates and capacities of in-home networks. Current day desktops and laptops are capable of generating data at rates of several hundred Mb/s to a few Gb/s that may need to be sent over a network. Correspondingly, the introduction of 802.11b has increased in-home network capacity to 11 Mb/s, and the more recent 802.11g further increases the capacity to 54 Mb/s (approximately OC-1 rate). Thus, we have a situation where core capacities and in-home data generation capacities have increased substantially, while the capacities of the intermediate network (viz., the last mile) have lagged far behind.

Figure 1 shows our envisioned access network architecture. We divide our access network architecture into three components: (a) Neighborhood network, (b) Distribution/Aggregation network, and (c) Metropolitan area network (MAN). The neighborhood network consists of a network of homes. The neighborhood network may evolve over the years through RF wireless, free-space optics, and fiber-based networks to provide higher data rates to the end user. The distribution/aggregation network connects neighborhoods to the MAN. We consider the MAN to be the core of the access network. The distribution/aggregation (DAN) network that connects the MAN and the neighborhood networks traditionally provide large bandwidth data, such as TV broadcast, to the end users. In addition, it plays a vital role in transferring data from the neighborhood network to the MAN. The focus of this paper is about information transfer from the optical access points (OAPs) in the neighborhood networks to a MAN node.

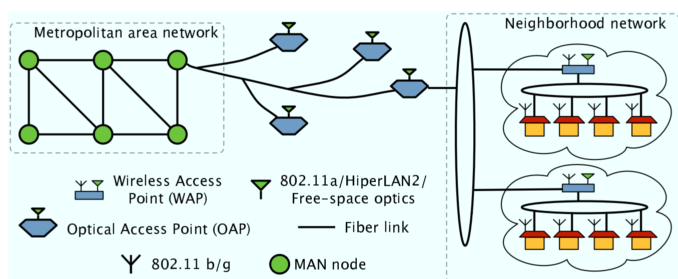

Fig. 1. Access network architecture.

Cable- and DSL-modem technologies form today's DAN network and has an uplink transmission rate of 12 Mb/s (using QPSK over a bandwidth of 6 MHz) or 24 Mb/s (using 16-QAM). The uplink data rate is shared by many regions, where each region serves thousands of home. The uplink data rate allocated for a region is about 2 Mb/s, shared by several homes in the region. Such a system is clearly not sufficient to support high-bandwidth applications such as video conferencing, interactive games, etc.

We assume a DAN based on optical communication where the OAPs form the end-nodes that transfer the aggregated data from the neighborhood networks. We employ Optical Code Division Multiple Access (OCDMA) as the access mechanism for the DAN. If the transmissions are perfectly synchronous (all users are aligned to bit boundaries), it is possible to design orthogonal codes. If the transmissions are asynchronous, the codes are designed such that the cross-correlation between any two codes are bounded under all shifts of the code. The number of such OCDMA codes for a given code length is significantly low.

We assume that the MAN node sends out synchronization pulses that the OAPs use to align their transmissions. As the propagation delay from the OAPs to the MAN may be different, we assume that the OAPs also employ “ranging” technique similar to that employed in today’s cable systems to estimate the propagation delay to the MAN node. Based on the measured propagation delay to the MAN node, the OAPs try to synchronize their transmissions to the bit boundaries. However, due to the variations in the measurement, the transmissions do not reach the MAN node at the same time. Figure 2 shows the transmissions from five different OAPs when it is received at the MAN node. The transmissions from the nodes are off from the bit boundary by a maximum of d units of time.

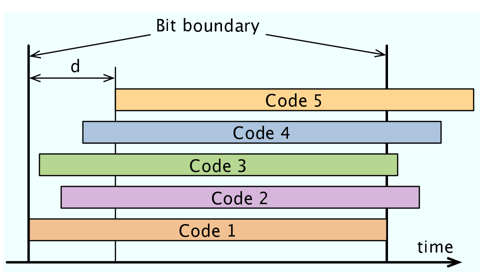

Fig. 2. Transmission from five different OAPs as received by the MAN node. The transmissions are off from the bit boundary by a maximum of d units of time.

The rest of the paper is organized as follows. Section II gives a brief background to the properties of OCDMA codes. Section III describes the construction of OCDMA codes for quasi-synchronous communication systems. Section IV discusses the receiver design and computation of bit error rate. Section V concludes the paper.

II. OCDMA Preliminaries

OCDMA divides the channel bandwidth across multiple users by assigning “codes”. The OCDMA codes are unipolar (binary numbers composed of 0s and 1s) unlike the traditional CDMA codes (that are bipolar composed of +1s and -1s). OCDMA transmission employs ON/OFF keying where the unipolar code is sent in place of 1 and no code is transmitted in place of 0. All the OCDMA codes employed are assumed to have k 1s. If the code length is v, then each user gets $\frac{1}{v}$ of the channel bandwidth.

If $x = (x_0, x_1, \ldots, x_{v-1})$ and $y = (y_0, y_1, \ldots, y_{v-1})$ denote two OCDMA codes, then the auto-correlation and cross-correlation are bounded by λ as shown below:

Auto-correlation:

$$\sum_{i=0}^{v-1} x_i x_{i \oplus \ell} \leq \lambda \qquad 1 \leq \ell \leq v-1 \tag{1}$$

Cross-correlation:

$$\sum_{i=0}^{v-1} x_i y_{i \oplus \ell} \leq \lambda \qquad 0 \leq \ell \leq v-1 \tag{2}$$

where $\oplus$ denotes modulo-v addition. The auto-correlation at shift 0 yields the weight of the codes k. The properties of the set of OCDMA codes are denoted by the three-tuple (v, k, λ). In optical systems, a set of codes with $\lambda = 1$ is referred to as *optical orthogonal codes* (OOCs).

Let s denote the number of chips of the OCDMA code transmitted during d units of time. If the transmissions are off from the bit boundaries by at most d units of time, it is sufficient that two users may be assigned a code that are orthogonal under shifts of up to s. In other words, the auto- and cross-correlation between any two codes employed in the system are bounded by λ for up to shifts of s.

Auto-correlation (quasi-synchronous):

$$\sum_{i=0}^{v-1} x_i x_{i \oplus \ell} \leq \lambda \qquad 1 \leq \ell \leq s \tag{3}$$

Cross-correlation (quasi-synchronous):

$$\sum_{i=0}^{v-1} x_i y_{i \oplus \ell} \leq \lambda \qquad 0 \leq \ell \leq s \tag{4}$$

We refer to OCDMA code sequences which are orthogonal for up to a certain maximum shifts as *finite shift-invariant* optical orthogonal codes.

III. Design of Finite Shift-Invariant OOC

We introduce the combinatorial objects used to design shift invariant OOCs. These objects have been extensively studied in the combinatorial literature, and some construction methods for them are described in [2], [3], [4], [5]. A balanced incomplete block design (BIBD) with parameters (v, k, λ) is an ordered pair (V, B), where V is a v-element set and B is a collection of b k-subsets[1] of V, called *blocks* such that every 2-subset of V is contained in exactly λ blocks. The notation BIBD(v, k, λ) specifies a BIBD on v points, with weight k, and index λ.

The point-block incidence matrix of a (V, B) design is a $v \times b$ matrix $A = [a_{ij}]$, in which $a_{ij} = 1$ if the i-th element of V occurs in the j-th block of B, and $a_{ij} = 0$ otherwise. An optical orthogonal code C is formed from columns of A. The column weight of A is k, which means that all spreading sequences have weight k, and the row weight of A is r, which means that r sequences have one at exactly the same position.

In this paper, we assume that $\lambda = 1$, because it implies that no more than one block contains the same pair of points. For $\lambda = 1$, no pair of columns of the matrix A contains two ones at the same positions. Equivalently, no pair of spreading sequences overlaps at more than one position. Recall that spreading sequences with this property are referred to as *orthogonal*. It is trivial to see that cyclic shifts of two orthogonal sequences are still orthogonal sequences. However if two sequences are cyclically shifted by different amounts, the corresponding sequences are in general not orthogonal any more. To ensure orthogonality under cyclical shift, a BIBD must be additionally constrained. A BIBD is *resolvable* if there exists a partition of its block set B into parallel classes, each of which partitions the set V. As we will show later, the resolvability is a key property necessary for the design of OOC codes.

Let V be a finite additive Abelian group of order v. Then t k-element subsets of V, $B_i = \{b_{i,1}, ..., b_{i,k}\},\ 1 \leq i \leq t$, create a (v, k, λ) difference family (DF) if every nonzero element of V occurs λ times among the differences $b_{i,m} - b_{i,n},\ 1 \leq i \leq t,\ 1 \leq m, n \leq k$. A set B_i is referred to as *base block*. If V is isomorphic to Z_v, the additive group of integers modulo v, then the corresponding (v, k, λ) DF is called a *cyclic difference family* (CDF).

If G is a group that acts on a set X, then the set $O_x = \{gx \mid g \in G\},\ x \in X$, is called the orbit of x. If G is a cyclic group of order v and X is the set of all base blocks of a CDF, a BIBD can be defined as the union of the orbits of X. The number of blocks in a BIBD is $b = tv$, where t is the number of base blocks. Block obtained from the same block form a resolvability class. Given a (v, k, λ) CDF with base blocks $B_i = \{b_{i,1}, ..., b_{i,k}\},\ 1 \leq i \leq t$, the point-block incidence matrix of the BIBD can be written in the form:

$$A = [A_1 | A_2 | \ldots | A_t] \qquad (5)$$

where each sub-matrix A_i is of dimension $v \times v$. The orbits of the base block B_j are represented by the positions of nonzero elements in the sub-matrix A_i. In other words, a column j in each sub-matrix A_i is a cyclic shift of the column $j - 1$ mod v.

Notice that any cyclic shift of any two different base blocks preserves orthogonality of the corresponding spreading sequences. This means that completely shift-invariant OOC can be obtained by taking first column from each sumbatrix $A_i,\ 1 \leq i \leq t$. However, the number of such sequences is small.

Consider now a relaxed constraint in which at most s shifts of the spreading sequences must be orthogonal. The number of corresponding sequences in one orbit is $\lfloor \frac{v}{s+1} \rfloor$, and the total number of shift-invariant OOC sequences is $N = \lfloor \frac{v}{s+1} \rfloor t$.

Example. The blocks $B_1 = \{0, 1, 4\}$ and $B_2 = \{0, 2, 7\}$ are the base blocks of a $(13, 3, 1)$ CDF of the group Z. The corresponding point-block incidence matrix is given by Equation 6.

If the maximum allowable shift is $s = 2$, then columns 1, 4, 7, and 10 of the two sub-matrices form orthogonal codes. The set of orthogonal codes (the columns of A) are written as matrix O where each row of the matrix represents a column in A.

$$O = \left[\begin{array}{ccccccccccccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \qquad (7)$$

The details of a CDF design is beyond the scope of this paper and interested readers can find more references in [2]. An example of a CDF class that is defined for a large number of block sizes, k, are prime power difference families [3], where v, the number of points is a prime. The number of base blocks is $t = \frac{v-1}{k(k-1)}$, and the total

[1] A k-subset is one in which every v-element has exactly k ones.

$$A = [A_1|A_2] = \left[\begin{array}{ccccccccccccc|ccccccccccccc}
1&0&0&0&0&0&0&0&0&1&0&0&1&1&0&0&0&0&0&1&0&0&0&0&1&0\\
1&1&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1\\
0&1&1&0&0&0&0&0&0&0&0&1&0&1&0&1&0&0&0&0&0&1&0&0&0&0\\
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0&0&0&0&0&0&0&0&1&0&0&1&1&0&0&0&0&0&1&0&0&0&0&1&0&1
\end{array}\right] \quad (6)$$

number of supported users is given by for a maximum shift of s is given by:

$$N = \left\lfloor \frac{v}{s+1} \right\rfloor \times t \quad (8)$$

As an illustration consider a CDF constructed by Wilson [5] using the group Z_v with $v = 151$ and weight $k = 6$. There are $t = 5$ base blocks: $B_1 = \{0, 6, 31, 70, 72, 117\}$, $B_2 = \{0, 48, 97, 107, 123, 30\}$, $B_3 = \{0, 82, 21, 101, 78, 89\}$, $B_4 = \{0, 52, 17, 53, 20, 108\}$, and $B_5 = \{0, 114, 136, 122, 9, 109\}$. To create OOC, every ($s$+1)-th block from each of t orbits is taken. Figure 3 shows the number of spreading sequences as a function of maximum allowable shift, s.

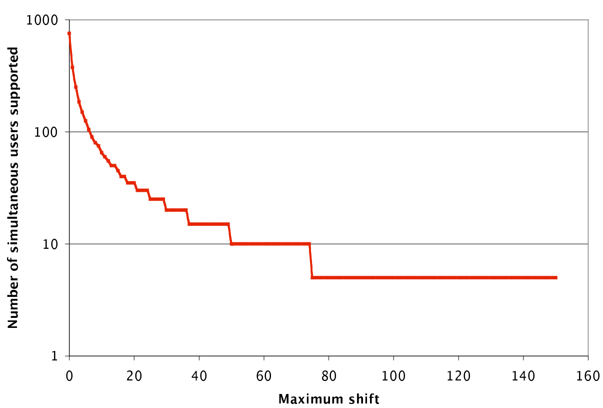

Fig. 3. Number of spreading sequences as a function of maximum allowable shift for Wilson construction (151, 6, 1).

Figure 4 shows the number of orthogonal codes, referred to as cardinality, of length v and weight k, obtained from prime power difference families.

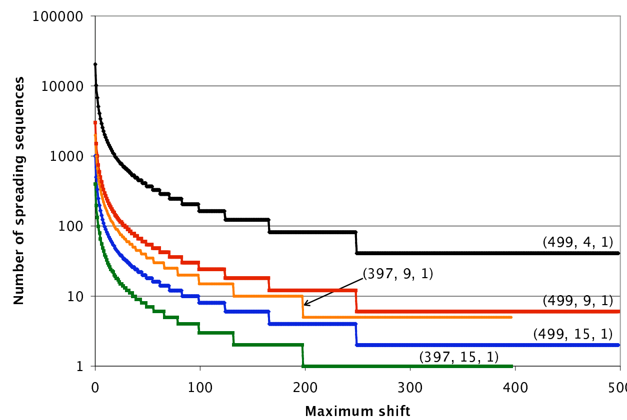

Fig. 4. Number of spreading sequences supported vs. maximum shift for different OOC parameters (v, k, λ).

IV. Decoder Design and BER Estimation

The decoder architecture is given in Figure 5. A_i^j denotes the j-th column in the submatrix A_i in the point-block incident matrix A that is chosen as a valid codeword. Notice that for synchronous ($s = 0$) or asynchronous case ($s = v - 1$), the decoder reduces to only one branch.

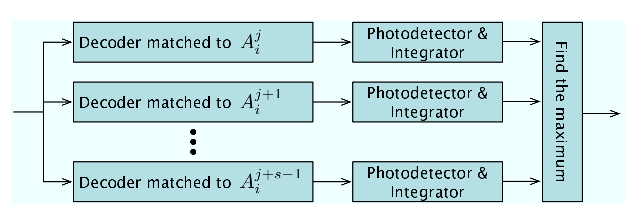

Fig. 5. Decoder architecture when the maximum shift is s.

We now compute the bit error rate of the quasi-synchronous codes. A similar approach is employed in estimating the BER performance of truncated Costas OOCs, proposed for asynchronous transmission [6].

In an incoherent optical CDMA systems the correlator output of the observed user j at time $t = T$ (T denotes the bit duration) is given by:

$$y = b_j w + I_j \tag{9}$$

where b_j denotes the transmitted bit, k denotes the codeword weight, and I_j denotes the multiuser interference. I_j is computed as

$$I_j = \sum_{n=1, n \neq j}^{N} I_n \tag{10}$$

where I_n denotes the interference contribution by the n-th user and N denotes the number of simultaneous users. The undesired signal is composed of $(N-1)$ interference signals each being the random variables. If I_n are independent and identically distributed, and if the number of user is sufficiently large, which is typically the case in practice, the resulting distribution will follow the Gaussian distribution according to the Central Limit Theorem.

Let $x = (x_0, x_1, \ldots, x_{v-1})$ and $y = (y_0, y_1, \ldots, y_{v-1})$ denote the codewords of two users. The cross-correlation function between them is computed as:

$$C_{xy}(l) = \sum_{i=0}^{v-1} x_i y_{i \oplus l} \qquad 0 \leq l \leq s \tag{11}$$

where $\oplus$ denotes a modulo-v addition. The variance of the cross-correlation is computed as:

$$\sigma_{xy}^2 = \frac{1}{s} \sum_{l=0}^{s} \left[C_{xy}(l) - \overline{C_{xy}} \right]^2 \tag{12}$$

where $\overline{C_{xy}}$ denotes the average cross-correlation amplitude over all possible shifts computed as:

$$\overline{C_{xy}} = \frac{1}{s+1} \sum_{l=0}^{s} C_{xy}(l) \tag{13}$$

In synchronous case, the maximum shift is $s = 0$ and the cross-correlation function reduces to in-phase cross-correlation, while in the asynchronous case the maximum allowable shift is $s = v - 1$. The mean (μ) and variance (σ^2) of multiuser interference (MUI) for N simultaneous users are computed as:

$$\mu = (N-1) \times \frac{1}{N(N-1)} \sum_{x=1}^{N} \sum_{y=1, y \neq x}^{N} \overline{C_{xy}} \tag{14}$$

$$= \frac{1}{N} \sum_{x=1}^{N} \sum_{y=1, y \neq x}^{N} \overline{C_{xy}} \tag{15}$$

$$\sigma^2 = (N-1) \times \frac{1}{N(N-1)} \sum_{x=1}^{N} \sum_{y=1, y \neq x}^{N} \sigma_{xy}^2 \tag{16}$$

$$= \frac{1}{N} \sum_{x=1}^{N} \sum_{y=1, y \neq x}^{N} \sigma_{xy}^2 \tag{17}$$

Using the Gaussian approximation for MUI and assuming that the system is MUI limited (neglecting the other sources of performance degradation), the bit-error rate (BER) for equiprobable mark- and space-state bits is computed as:

$$\text{BER} = \frac{1}{4} \text{efrc} \left(\frac{\mu_1 - t_{sh}}{\sigma \sqrt{2}} \right) + \frac{1}{4} \text{efrc} \left(\frac{t_{sh} - \mu_0}{\sigma \sqrt{2}} \right) \tag{18}$$

where t_{sh} is the threshold, and μ_0 and μ_1 are the mean values of space- and mark-state bits. μ_0 and μ_1 are computed as:

$$\mu_0 = \mu \tag{19}$$

$$\mu_1 = \mu + k \tag{20}$$

erfc(.) is the complementary error function defined as:

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} \, du. \tag{21}$$

When the optimum threshold is set to $\frac{k}{2} + \mu$, the BER reduces to:

$$\text{BER} = \frac{1}{2} \text{erfc} \left(\frac{\frac{k}{2}}{\sigma \sqrt{2}} \right) \tag{22}$$

Figure 6 shows the BER performance as a function of number of simultaneous users for different allowable shifts. The curve $s = 0$ corresponds to the synchronous case and the curve $s = v - 1 = 150$ corresponds to the asynchronous case. In the synchronous case, we are able to support 755 users, while in asynchronous case only 5. The implementation complexity, however, decreases as s increases. It is observed that for a given BER, the number of users supported may be increased by a factor of $\left\lfloor \frac{v}{s+1} \right\rfloor$.

V. Conclusion

In this paper, we demonstrated that the number of users supported by an OCDMA system may be increased by a factor of $\lfloor \frac{v}{s+1} \rfloor$, if the transmissions are quasi-synchronous where the transmissions are off by at most s chip transmission duration. The approach developed in this paper is to derive OCDMA codes from those that are orthogonal under all shifts. It is not known if the number of users supported under this approach is the maximum possible or not.

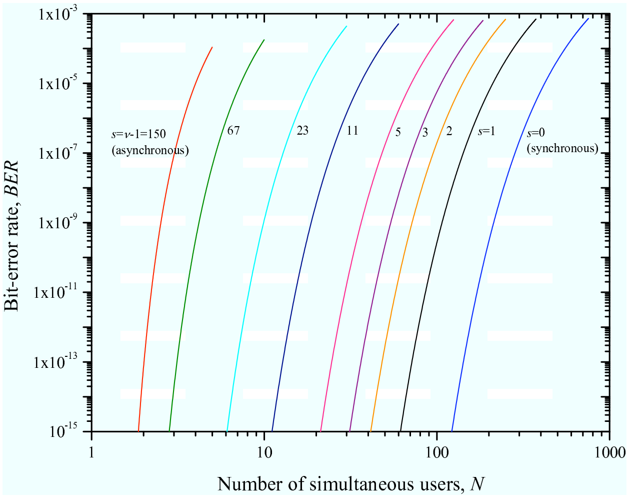

Fig. 6. Bit error rate as a function of number of simultaneous users for Wilson (151, 6, 1) construction.

ACKNOWLEDGMENT

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