Coverage Time Characteristics in Sensor Networks

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***Abstract*— We study the problem of coverage of a given area for a maximum duration using a set of battery-operated sensors. Each sensor has a fixed sensing range and a limited lifetime due to the finite battery capacity. Sensors can be activated and deactivated at any time. The goal of this paper is to a find a schedule, determining when to activate and deactivate each sensor, to maximize the time for which every point in the area is covered by at least one sensor. We present several algorithms for this problem and show experimental and theoretical evidences to their efficiency. We also present an algorithm for a new model of coverage, called *weak coverage*, that does not require each point of the region to be covered all times, as long as the regions that are not covered are small.**

I. Problem Formulation

Consider a sensor network defined by a set of sensors $\mathcal{S}$ and a set of points $\mathcal{P}$ to be covered. Let $\mathcal{P}_s$ denote the set of points covered by a sensor and $\mathcal{S}_p$ denote the set of sensors that cover a point p. A *k-redundant cover* (or simply a k-cover) is a collection of sensors such that every point is covered by at least k sensors. A cover C is said to be *minimal k-cover* if $\mathcal{C}$ is a k-cover and $\mathcal{C} \setminus \{s\}$ is not a k-cover for any $s \in \mathcal{C}$. The objective of a network is to maintain k-coverage of all the points. The lifetime of the network, L_k is defined as the maximum time for which k-coverage can be maintained.

Definitions: The *depth of a point p*, denoted by d_p, is defined as the number of sensors that cover the point, i.e. $d_p = |\mathcal{S}_p|$. The *depth of a sensor s*, denoted by d_s, is defined as the minimum depth of a point in its neighborhood, i.e. $d_s = \min_{p \in \mathcal{P}_s} d_p$. The *depth of the problem*, denoted by (d), is defined as the minimum number of sensors that cover any point in the region, i.e. $d = \min_{p \in \mathcal{P}} d_p$.

Different techniques have been proposed in [1], [2] and [3] for maximizing the lifetime by allowing certain nodes to sleep. In [4], [5] and [6], energy efficient centralized mechanisms are developed by dividing the sensor nodes into disjoint sets such that each set completely covers the region. The goal of these approaches is to determine the maximum number of disjoint sets.

In [7], it is observed that employing non-disjoint covers may result in higher lifetime than disjoint covers. According to their results, the optimal lifetime of the network is upper-bounded by a factorIn [8], the authors derive an upper-bound on the network lifetime achievable by any algorithm when only a fraction α of the region needs to be covered. For $\alpha = 1$, their result yields the bound $L^* \leq d$. Therefore, the bound in [**?**] is conservative as $d_{\max}$ may be much larger than d.

The depth of the problem clearly defines the upper bound on the coverage time of the network. The optimal coverage time may be strictly less than the depth in the 2-dimensional case. For example, consider a rectangular region that needs to be covered using five sensors. The areas covered by five sensors are shown in Figure 1. It may be observed that every point in the region is covered by two sensors. However, the maximum achievable coverage time in the network is only 1.5 assuming that every sensor has one unit of lifetime. The network has three possible covers, $\{1, 2\}$, $\{1, 3, 4\}$, and $\{2, 3, 5\}$, and the maximum network lifetime is achieved when each of these covers is operated for 0.5 units of time. It is worth noting that disjoint cover selection would only result in a coverage time of one unit.

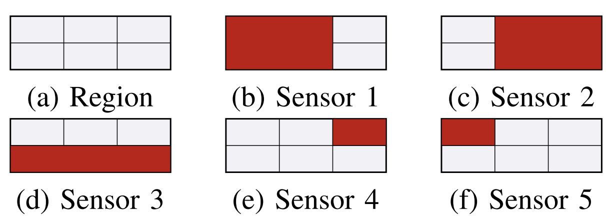


Fig. 1. A set of five sensors and their coverage regions. The region to be covered is divided into grids of two rows and three columns. The region is said to be covered if the centers of all the grids are covered.

Let $\mathcal{M}$ denote the set of all possible minimal k-covers in the network. Let $|\mathcal{M}|$ denote the number of minimal

k-covers. In order to maintain k-coverage at all times, a sequence of covers from $\mathcal{M}$ needs to be employed, each for a certain duration of time. Let $x = [x_i]_{|\mathcal{M}|\times 1}$ be a column vector that denotes the usage time of the minimal covers. Let $A = [a_{ij}]_{|\mathcal{S}|\times|\mathcal{M}|}$ be a matrix where an element a_{ij} denotes sensor i is present in cover j or not; 1 if true, 0 otherwise. Let $b = [b_i]_{|\mathcal{S}|\times 1}$ be a column vector that represents the maximum lifetime of the sensors. Let $u = [1]_{|\mathcal{M}|\times 1}$ be a unit column vector.

The maximum lifetime of the network under k-coverage may be modeled as a linear program as below:

$$\begin{array}{ll} \texttt{MAXCOVERAGE}: & \textbf{Maximize } L_k = u^T x \\ \textbf{subject to:} & Ax \leq b \\ & x \geq 0 \end{array}$$

The `MAXCOVERAGE` problem has an optimal solution as the maximum lifetime of the network is bounded. For example, the maximum lifetime under 1-coverage is bounded by $\min_p \left(\sum_{s\in\mathcal{S}_p} b_s\right)$. While the above formulation may provide the maximum coverage time, the solution time may not be practical as the set of all possible minimal covers is not P-enumerable. Note that the number of rows in the above formulation is $|\mathcal{S}|$, hence there cannot be more than $|\mathcal{S}|$ linearly independent columns in A. Therefore, maximum coverage time may be achieved with at most $|\mathcal{S}|$ covers when the covers can be operated for different durations of time.

A. Computing optimal solution with reduced complexity

The maximum coverage time may be obtained with reduced complexity by considering the dual of the `MAXCOVERAGE` problem, written as:

$$\begin{array}{ll} \texttt{MAXCOVERAGE}-\texttt{DUAL}: & \textbf{Minimize } b^T y \\ \textbf{subject to:} & A^T y \geq u \\ & y \geq 0 \end{array}$$

The dual formulation consists of $|\mathcal{S}|$ variables and potentially exponential number of constraints. Note that every constraint (row in A^T) is a possible minimal cover. Instead of solving the dual problem with all possible minimal covers, we solve it iteratively by adding one constraint at a time. The steps of the iterative approach is shown in Figure 2.

The complexity of the iterative approach is determined by the time taken to identify a minimal cover $\mathcal{C}$, such that $y(\mathcal{C}) = \sum_{s\in C} y_s < 1$. To identify such a cover, we identify the minimal cover $\mathcal{C}$ such that $y(\mathcal{C})$ is minimum among all minimum covers, referred to as the minimum weighted sensor cover (MWSC). If the minimum $y(\mathcal{C}) \geq 1$, then the solution of the dual problem gives the maximum coverage time.

1) **Step 1.** Initialize $y_s = 0,\ \forall s \in \mathcal{S}$.
2) **Step 2.** Initialize the dual problem with no constraints.
3) **Step 3.** Obtain the solution for the dual problem with the current constraint set.
4) **Step 4.** Identify if there is a possible violation of any of the constraints (among all possible covers), i.e. identify a minimal cover $\mathcal{C}$ such that $\sum_{s\in\mathcal{C}} y_s < 1$. If such a cover cannot be obtained, then the solution is confirmed as optimal, go to Step 6.
5) **Step 5.** Add the constraint corresponding to the minimal cover $\mathcal{C}$ and go to Step 3.
6) **Step 6.** Stop.

Fig. 2. Iterative approach to compute the maximum coverage time.

As an alternative to obtain MWSC by searching through all possible minimal cover, whose complexity is $O(2^{|\mathcal{S}|})$, we develop an approach whose complexity is $O(2^{\sqrt{|\mathcal{S}|}})$. Assume that the region to be covered is rectangular. Divide the region to be covered into two halves through a vertical line. Let $\mathcal{S}_d$ denote the set of sensors whose coverage area intersect the line; $\mathcal{S}_l$ denote the set of sensors whose coverage area lie to the left of the line, and $\mathcal{S}_r$ denote the set of sensors whose coverage area lie to the right of the line. Note that $\mathcal{S}_d$, $\mathcal{S}_l$, and $\mathcal{S}_r$ are mutually disjoint. Similarly, any minimal cover $\mathcal{C}$ can be split into three mutually disjoint components, $\mathcal{C}_d$, $\mathcal{C}_l$, and $\mathcal{C}_r$. Hence, $y(\mathcal{C}) = y(\mathcal{C}_d)+y(\mathcal{C}_l)+y(\mathcal{C}_r)$. Minimizing $y(\mathcal{C})$ implies minimization of each of the term in the summation. $y(\mathcal{C}_l)$ and $y(\mathcal{C}_r)$ are dependent on $y(\mathcal{C}_d)$, however $y(\mathcal{C}_l)$ and $y(\mathcal{C}_r)$ are independent of each other for any given value of $y(\mathcal{C}_d)$. Computing the MWSC requires consideration of all possible combinations of the sensors in $\mathcal{S}_d$.

Complexity analysis. Given a set of $\mathcal{S}$ sensors in a two-dimensional region distributed uniformly, the number of sensors that lie on the dividing line is on the order of $\sqrt{|\mathcal{S}|}$. The number of sensors on the left and right hand side of the region is approximately $\frac{|\mathcal{S}|}{2}$. Therefore, the complexity of computing the MWSC may be written recursively as:

$$2^{\sqrt{|\mathcal{S}|}} \times 2 \times \left[2^{\sqrt{\frac{|\mathcal{S}|}{2}}} \times 2 \times \left[2^{\sqrt{\frac{|\mathcal{S}|}{4}}} \times 2 \times \ldots\right.\right.$$

$$= O(2^{\sqrt{|\mathcal{S}|}\ +\ \log|\mathcal{S}|}) = O(2^{\sqrt{|\mathcal{S}|}})$$

Although the above complexity is significantly lower than exponential in $|\mathcal{S}|$, it is still impractical to compute the maximum achievable coverage time for large networks.

II. Maximum Coverage Time Under Phased Operation

A practical way to cover a region is to operate over a finite number of phases. A minimal cover is chosen to be the active set during a phase. We consider the problem of maximizing the lifetime of a sensor network under the constraint that the number of phases employed is R and each phase must be operated for the same duration of time t. The coverage time of the network when operated in R phases, denoted by L_R is given by $L_R = Rt$. Let $z = [z_i]_{|\mathcal{M}|\times 1}$ be a column vector that represents the number of phases for which a cover is chosen. Clearly, $\sum_{i=1}^{|\mathcal{M}|} z_i = R$.

The formulation of the above problem and its modification defined as the $\texttt{MAXINTERVAL}$ and $\texttt{MAXINTERVAL}-2$ problem is shown in figure 3(a) and 3(b). We observe that the optimal value of t, say t^*, must be such that at least one of the constraints must be met with equality. Let $e = [e_i]_{|\mathcal{S}|\times 1} = Dz$ be a column vector and e_{max} denote the maximum element in the column vector e, then $t^* = \frac{1}{e_{max}}$. The $\texttt{MAXINTERVAL}-2$ may be re-written with an objective to maximize phase duration as shown in Figure 3(c) called the $\texttt{MAXINTERVAL}-3$ problem.

The $\texttt{MAXINTERVAL}-3$ provides the intuition behind selecting a cover during a phase. The minimal cover to be selected during a phase must be such that the most energy spent by a sensor in the minimal cover must be minimized, referred to as the *Min-Max* heuristic. Thus, we order the sensors at each phase, denoted by $\mathcal{S}'$, based on the decreasing order of their active time (number of phases in which the sensor was chosen) and check whether the removal of sensor $s \in \mathcal{S}'$ (selected in order) would violate the cover, if yes it is made active otherwise inactive. At every iteration we select a minimal cover.

Figure 4 shows the achievable lifetime as a function of the number of phases when the phases are operated with equal (heuristic) and unequal (dual program) durations. The lifetime with unequal phase times is obtained by solving the dual program with only the covers selected by the Min-Max heuristic. It is observed that the achievable coverage time is not strictly an increasing function. We observe that the lifetime converges to a value asymptotically. While we observe that the value is optimal for small networks (by comparing with the solution from linear programs), a theoretical proof for the convergence to optimal solution is not available as of this writing.

The above results highlight a method to select the number of phases to be employed. Assume that the sensors have same (unit) lifetime. Let R_m denote the number of phases operated such that no sensor is present in more than m phases. Since a sensor is present in at most m phases, the coverage time of the network under equal phase time is $\frac{R_m}{m}$. If one more phase is employed, then there is at least one sensor that will be present in $m+1$ phases, hence the lifetime is $\frac{R_m+1}{m+1} \leq \frac{R_m}{m}$. Therefore, the number of phases to be employed is determined by choosing an appropriate value for m and selecting as many covers as possible such that no sensor is present in more than m covers and employing each cover for $\frac{1}{m}$ units of time.

III. Fuzzy Coverage

While coverage of the entire region is desired, it is not practical to exactly characterize the boundary of a sensor's range. We can take advantage of this fact by assuming that rather than checking if the whole region is covered, we check only if the vertices of a grid Γ are covered, where every cell of Γ is of size $\varepsilon \times \varepsilon$. We consider a greedy algorithm that can be implemented in a distributed manner. We generate a minimal layer $L_1 \subseteq \mathcal{S}$, remove it from $\mathcal{S}$, and repeat.

We present two theoretical results showing that in this case the number of layers obtained is up to a constant factor (that depends on ε) optimal. The approximation factor depends on the sensing shape that each sensor forms. We study the case where the sensing range is an axis-parallel square, and the case where the sensing range is a unit disk. In either case, we say that $L \subseteq \mathcal{S}$ is a *fuzzy cover* if it covers every vertex of the grid of Γ. Needless to say that these bounds only constitutes lower bounds, and our experiments show that in practice the actual quality of the approximations is significantly better.

Let $\{L_1^* \dots L_{k*}^*\}$ be a partition of $\mathcal{S}$ into a maximum number of layers, each is a fuzzy cover. Let $\{L_1 \dots L_{k_{app}}\}$ be any partition of $\mathcal{S}$ into layers, such that each L_i is a minimal fuzzy cover, and the set $\mathcal{S} \setminus (L_1 \cup \dots \cup L_{k_{app}})$ is not a fuzzy cover.

Theorem *3.1:* If the sensing range of the sensors of $\mathcal{S}$ are unit axis-parallel squares, then $k_{app} \geq (\varepsilon/2)k^*$. If they are unit disks, then $k_{app} \geq (\varepsilon/12)k^*$

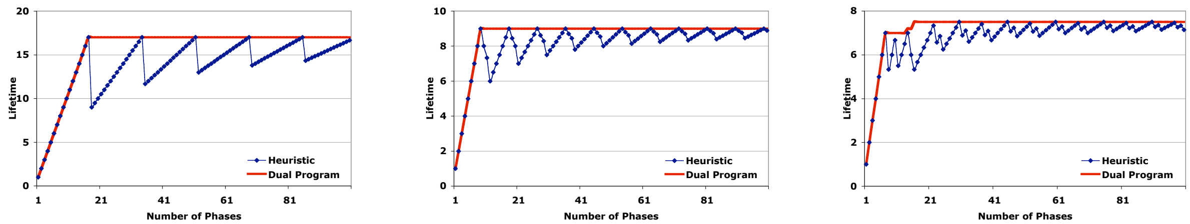
The proof is omitted due to space constraints.

IV. κ-weak cover

In surveillance applications, sometimes it is not necessary that every point in the region is covered, as long as the regions which are not covered are not "too large." We formulate this notion by defining "weak coverage,"

$\texttt{MAXINTERVAL}:$ **Maximize** $L_R = Rt$	$\texttt{MAXINTERVAL}-2:$ **Maximize** $L_R = Rt$	$\texttt{MAXINTERVAL}-3:$ **Minimize** $\max_i e_i$
subject to: $u^T z = R$	**subject to:** $u^T z = R$	**subject to:** $u^T z = R$
$Azt \leq b$	$Dzt \leq v$	$e = Dz$
$z = \{0, 1, 2, \ldots\}$, $t \geq 0$	$z = \{0, 1, 2, \ldots\}$	$z = \{0, 1, 2, \ldots\}$
(a)	(b)	(c)

Fig. 3. Linear programming formulations for maximum coverage time under phased operation.



(a) (b) (c)

Fig. 4. Coverage time versus number of phases employed for a sensor network of size 10×10 with a sensor coverage range of 5×5: (a) 100 sensors each with lifetime of 1 unit; (b) 50 sensors each with a lifetime of 1 unit; and(c) 25 sensors with lifetime chosen as {1, 2, 3, 4, 5} uniformly

and show that a greedy algorithm for computing such covers yields a constant-factor approximation.

Let $\mathcal{S}$ be a set of n axis-parallel unit squares, each represents the sensing region of a single sensor. Let $\kappa > 1$ be a fixed parameter. We say that a pair of points p, q is a κ-pair if the Euclidean distance between them is κ. We say that a curve μ is a κ-curve if its endpoints form a κ-pair. Let R be a region in the plane. We say that $L \subseteq \mathcal{S}$ is a κ-*weak cover* if every κ-curve that is contained in R intersects a square of L. We assume that $\kappa > 1$ is a constant. We present a greedy algorithm for partitioning $\mathcal{S}$ into layers, so that each layer is a κ-weak cover, and the number of layers, is up to a constant, optimal. We say that $L \subseteq \mathcal{S}$ is a *minimal* κ-weak cover if it is a κ-weak cover, but removing any square would violate this property.

Analogous to the previous section, let $\{L_1^* \ldots L_{k*}^*\}$ be a partition of $\mathcal{S}$ into a maximum number of layers, where each layer is a κ-weak cover. Let $\{L_1 \ldots L_{k_{app}}\}$ be any partition of $\mathcal{S}$ into layers, such that each L_i is a minimal κ-weak coverand the set $\mathcal{S} \setminus (L_1 \cup \ldots \cup L_{k_{app}})$ is not κ-weak cover.

Theorem *4.1:* There a constant $c_2 > 0$ such that $k_{app} \geq k^*/c_2$.

The proof is omitted due to space constraints.

V. Conclusion

This paper develops an effective schedule to activate and deactivate sensors to maintain full coverage with an objective to increase the lifetime of the network. An exact method of computing the optimal solution with a significant lower computational complexity and an iterative approach of selecting covers so as to increase network lifetime is also developed. A distributed greedy algorithm for selecting covers is developed for a new model of coverage, called the weak coverage, and is shown to yield a constant-factor approximation.

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