Analyzing Channel Assignment with Rearrangement in Multi-Channel Wireless Line Networks

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Abstract— **In this paper, we analyze the blocking performance of a channel assignment scheme in a multi-channel wireless line network. We assume that the existing calls in the network may be rearranged on different channels to accommodate an incoming call. The analysis is limited to single-hop calls with different transmission ranges. Through extensive simulations, we demonstrate that the developed analytical model closely approximates the values obtained through simulations.**

I. INTRODUCTION

A wireless mesh network (WMN) is a densely deployed multi-hop wireless network [1]. All nodes in the network have routing capabilities and are connected to each other (via multiple hops if destination node is not present in the communication range of the source node). These nodes are generally not mobile and therefore topology changes are not frequent. Due to the high connectivity and static nature, WMNs offer a resilient and flexible wireless infrastructure. WMNs have a wide area of applications, including PORTAL, public internet access, video streaming, and underground mining [1], [2], [3], [4]. These networks are self-organizing in nature and can reconfigure themselves in case of a failure of a node. WMNs provide low-cost scalable networks and support broadband data. Recently, WMNs have gained attention due to advancements in the field of smart antennas and software-controlled radio transceivers.

The scarcity of wireless bandwidth is a major motivation to achieve efficient bandwidth utilization. To improve throughput of a WMN, nodes may be equipped with multiple wireless interface cards to allow simultaneous transmission/reception over orthogonal channels. Such networks are referred to as *multi-channel wireless mesh networks* (MC-WMNs). The operation of such MC-WMNs typically involves identifying a free channel for a call from a given source to destination. The channel utilization in the network may be improved by employing efficient channel assignment strategies. The major factor that affects channel utilization is the interference constraint – i.e., a node may not receive on the same channel from two different nodes; and a node may not transmit and receive on the same channel at any time.

Various channel assignment schemes have been proposed in the literature for WMNs employing omnidirectional antennas. For a given set of calls and traffic load information, the channel assignment problem may be mapped to the Distance-2 coloring[1] problem [5], [6], [7], [8], [9], [10], [11] with a constraint on the number of transceivers at each node. Distance-2 coloring problem is a well known NP-hard problem. Various approximation algorithms and heuristics have been developed for the channel assignment in above networks with different objectives, such as minimizing interference, maximizing throughput, and minimizing the number of channels required. In [12], the authors adopt an approach based on balanced incomplete block design to assign channels for each interface card such that the communication network is 2-edge-connected and the interference is minimum. In [13], the authors develop a heuristic based on random channel assignment policy for channel assignment in a distributed manner while maintaining connectivity of the network.

The capacity of the network can be improved further by employing directional antennas [14], [15], [16]. Directional antenna provides spatial separation between transmissions and increases spatial reuse. In [17], [18], the authors map the problem of channel assignment/link scheduling with directional antenna to the edge-coloring problem, i.e., the problem to color the edges of a graph such that no two adjacent edges share the same color.

In this paper, we consider the rearrangement channel assignment scheme in MC-WMNs. Rearrangement channel assignment scheme accepts a call if resources are available, even if it requires rearrangement of channels for existing calls. It was first proposed in [19] in the context of cellular networks as maximum packing scheme. In [20], Zafer et. al. analyze the rearrangement scheme for a single channel wireless line network employing omnidirectional antennas. They develop an analytical model to evaluate blocking probability and study the impact of transmission range and hop length. However, their model is restricted to a single-channel network.

In this paper, we develop an analytical model for channel assignment with rearrangement in multi-channel wireless line networks. We model the effect of both directional and omnidirectional antennas for multiple channels. We evaluate the effectiveness of the developed analytical model by comparing the results to those obtained using simulations. The rest of the paper is organized as follows. In Section II, we explain the network model and interference constraints when employing

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[1] The problem of coloring the vertices/edges of a graph such that no two vertices/edges of at most two hops away from each other share the same color.

omnidirectional and directional antennas. In Section III, we explain the rearrangement channel assignment scheme and develop the necessary and sufficient condition that guarantees a valid channel assignment under this scheme for a line network. In Section IV, we analyze the blocking probability of a call in multi-channel line networks employing omnidirectional and directional antennas. Section V demonstrates the effectiveness of the developed analytical model. We conclude the paper in Section VI.

II. SYSTEM MODEL

Consider a multi-channel wireless line network where nodes are placed along the x-axis on integer coordinates. Assume that the nodes are numbered using integers corresponding to its x-coordinate and that there are countably infinite number of nodes in the network. Let C denote the number of orthogonal channels in the network. Every node is assumed to be equipped with C transceivers. The transmission and interference range of every node is assumed to be R units.

We consider two scenarios where all the nodes in the network employ either: (a) omnidirectional antennas; or (b) directional antennas. In case of nodes employing omnidirectional antenna with a transmission range of R, the transmission of node i on channel c is received by node j if and only if: (1) $|j-i| \leq R$; and (2) no node in the range $[j-R, j+R]$ (including node j) transmits on channel c. In case of nodes employing directional antenna with a transmission range of R, the transmission of node i on channel c directed along the positive x-axis is received by node j if and only if: (1) $1 \leq j-i \leq R$; (2) no node in the range $[j-R, j]$ transmits on channel c along the positive x-axis; and (3) no node in the range $[j, j+R]$ transmits on channel c along the negative x-axis. Similarly, the transmission of node i on channel c directed along the negative x-axis is received by node j if and only if: (1)$1 \leq i-j \leq R$; (2) no node in the range $[j, j+R]$ transmits on channel c along the negative x-axis; and (3) no node in the range $[j-R, j]$ transmits on channel c along the positive x-axis.

We assume that the call arrival process at a node follows a Poisson process with rate λ and the holding time follows an exponential distribution with rate μ. Let $\rho = \lambda/\mu$. A call originating at node i is assumed to be always destined to node $i+R$ (single-hop calls). Every call requires one channel worth of bandwidth. The calls are assumed to be bidirectional and the source and destination share the channel assigned by the channel assignment algorithm to communicate with each other. We assume that there can be multiple calls established between a source and destination, however they have to be assigned distinct channels.

III. REARRANGEMENT CHANNEL ASSIGNMENT

Given the network status (the channels assigned to calls in progress), and a new call request that arrives at node i, the objective is to compute if a channel can be assigned to the call or not without violating the interference constraints. In order to improve the efficiency of channel assignment, we assume that the existing calls may be rearranged (to different channels) in order to accommodate the new call.

Assume that a new call request arrives at node i at some time instant. Let X_j denote the number of calls originated at node j that are in progress at that time instant. In the following sections, we derive the necessary and sufficient conditions on the network status to accommodate a new call originating at node i when the nodes employ omnidirectional and directional antennas.

A. Employing omnidirectional antennas

The call requires a channel to be assigned to enable nodes i and $i+R$ to communicate with each other. Let V_k denote the sum of the calls originating from nodes in the range $[k, k+2R]$, computed as:

$$V_k = \sum_{j=k}^{k+2R} X_j \tag{1}$$

We refer to the set of nodes involved in the computation of V_k as a "V-window" starting at node k. Observe that every call originating within a V-window must be assigned a unique channel for the calls to be successful. It may be shown that a channel for the new call request at node i may be assigned by rearranging the existing calls if the following condition is satisfied.

$$V_k < C, \quad \forall k \in \{i-2R, i\}, \tag{2}$$

The above condition implies that for every V-window that includes the new call request, total calls originating from nodes in the V-window should be less than C. For example, when $R=1$, the size of a V-window is 3. Therefore, for a new call originating at node i, we require V_{i-2}, V_{i-1}, and V_i each to be less than C.

Note that the above is a necessary condition. However, we may show that is also sufficient through induction. We observe that in any V-window, the number of calls originating is always less than C. If we consider two consecutive V-windows, say V_k and V_{k+1}, the channels used by node k may be reused by node $k+2R+1$. As the total number of calls in any V-window remains below C, there is always sufficient number of channels available to assign for calls originating from node $k+2R+1$. By applying the above argument over successive V-windows, it may be shown that the above condition is also sufficient.

B. Employing directional antennas

Let W_k denote the sum of the calls originating from nodes in the range $[k, k+R]$, computed as:

$$W_k = \sum_{j=k}^{k+R} X_j \tag{3}$$

We refer to the set of nodes involved in the computation of W_k as a "W-window" starting from node k. Again, observe that every call originating within a W-window must be assigned a unique channel for them to be successful. It may be shown that a channel for the new call request at node i may be assigned by rearranging the existing calls if the following condition is satisfied.

$$W_k < C, \quad \forall k \in \{i - R, i\} \tag{4}$$

For example, when $R = 1$, the size of a W-window is 2. Therefore, for a new call originating at node i, we require W_{i-1} and W_i each to be less than C.

The proof of the necessity and sufficiency of the above condition is similar to that of the omnidirectional antenna scenario with only the size of the window changed from $2R+1$ to $R+1$.

IV. BLOCKING PROBABILITY ANALYSIS FOR REARRANGEMENT CHANNEL ASSIGNMENT

In this section, we develop the analytical model to compute the blocking probability of a call under channel assignment policy with rearrangement in a line network employing omnidirectional and directional antennas.

A. Employing omnidirectional antennas

To compute the blocking probability of a call request that arrives at node i at a time instant, we compute the probability that the necessary and sufficient condition as specified in Equation 2 is not satisfied. In order to compute this probability, we compute several joint and conditional probabilities as below.

Consider two consecutive V-windows starting from nodes k and $k+1$. We seek to compute $P[V_{k+1} = v_{k+1} | V_k = v_k]$. Let $Y_k = V_k - X_k$. The joint distribution of X_k and Y_k may be viewed as a 2-dimensional Markov Chain and the steady-state joint distribution $P(X_k, Y_k)$ may be obtained as below.

$$P(X_k = x_k, Y_k = y_k) = \frac{1}{G_o} \frac{\rho^{x_k} (2R\rho)^{y_k}}{x_k! y_k!} \tag{5}$$

where $0 \leq x_k \leq C$, and $0 \leq y_k \leq C - x_k$. G_o denotes the normalization constant given by:

$$G_o = \sum_{x_k=0}^{C} \sum_{y_k=0}^{C-x_k} \frac{\rho^{x_k} (2R\rho)^{y_k}}{x_k! y_k!} \tag{6}$$

From the above joint probability, $P(V_k = v_k)$ is computed as:

$$P(V_k = v_k) = \sum_{x_k=0}^{v_k} P(X_k = x_k, Y_k = v_k - x_k) \tag{7}$$

$P(X_k = x_k)$ and $P(Y_k = y_k)$ are computed as:

$$P(X_k = x_k) = \sum_{y_k=0}^{C-x_k} P(X_k = x_k, Y_k = y_k) \tag{8}$$

$$P(Y_k = y_k) = \sum_{x_k=0}^{C-y_k} P(X_k = x_k, Y_k = y_k) \tag{9}$$

The conditional probability $P(Y_k = y_k | V_k = v_k)$ is computed as:

$$P(Y_k = y_k | V_k = v_k) = \frac{P(X_k = v_k - y_k, Y_k = y_k)}{P(V_k = v_k)} \tag{10}$$

The conditional probability $P(X_k = x_k | Y_k = y_k)$ is computed as:

$$P(X_k = x_k | Y_k = y_k) = \frac{P(X_k = x_k, Y_k = y_k)}{P(Y_k = y_k)} \tag{11}$$

From the above probabilities, we compute the conditional probability of the number of calls originating in one conditional window given the number of calls originating in the previous window as:

$$P(V_{k+1} = v_{k+1} | V_k = v_k) = \sum_{y_k=0}^{v_k} P(Y_k = y_k | V_k = v_k) P(X_{k+2R+1} = v_{k+1} - y_k | Y_k = y_k) \tag{12}$$

Due to the symmetrical nature of the network, it is reasonable to assume that the distribution of X_k is identical for all k. Therefore, the above probability may be re-written as:

$$P(V_{k+1} = v_{k+1} | V_k = v_k) = \sum_{y_k=0}^{v_k} P(Y_k = y_k | V_k = v_k) P(X_k = v_{k+1} - y_k | Y_k = y_k) \tag{13}$$

From the above conditional probability, we compute the steady-state distribution of the number of calls in a V-window, denoted by $\pi = (\pi_0, \pi_1, \ldots, \pi_C)$, where π_c denotes the steady-state probability that $V_k = c, 0 \leq c \leq C$. The steady-state probability distribution is obtained by solving the equations $\pi P(V_{k+1} = v_{k+1} | V_k = v_k) = \pi$ and $\sum_{c=0}^{C} \pi_c = 1$.

Now we derive the probability that no call originating in a V-window is blocked given that no call originating in the previous window is blocked. It may be written as:

$$\begin{aligned} & P(V_{k+1} < C | V_k < C) \\ &= \sum_{v_k=0}^{C-1} P(V_k = v_k | V_k < C) P(V_{k+1} < C | V_k = v_k) \\ &= \sum_{v_{k+1}=0}^{C-1} \sum_{v_k=0}^{C-1} P(V_k = v_k | V_k < C) \times \\ &\qquad P(V_{k+1} = v_{k+1} | V_k = v_k) \\ &= \sum_{v_{k+1}=0}^{C-1} \sum_{v_k=0}^{C-1} \frac{P(V_k = v_k)}{1 - P(V_k = C)} \times \\ &\qquad P(V_{k+1} = v_{k+1} | V_k = v_k) \end{aligned} \tag{14}$$

Note that due to the symmetry of the network, we may assume that all of the above derived probabilities are identical for any value of k. From the above probabilities, we may compute the probability that Equation 2, denoted by P_{so} is satisfied is given by:

$$\begin{aligned} P_{so} &= \pi(V_{i-2R} < C) \prod_{k=i-2R}^{i-1} P(V_{k+1} < C | V_k < C) \\ &= \pi(V_{i-2R} < C) P(V_{k+1} < C | V_k < C)^{2R} \end{aligned} \tag{15}$$

The blocking probability for the line networks employing omnidirectional antennas is given by:

$$P_{bo} = 1 - P_{so}. \tag{16}$$

B. Employing directional antennas

In case of directional antennas, for any given call request between a source node s and destination node d, it is sufficient to satisfy Equation 4 for $R+1$ consecutive W-windows. Let $Z_k = W_k - X_k$. The steady-state joint distribution $P(X_k, Z_k)$ may be given as below:

$$P(X_k = x_k, Z_k = z_k) \;\; = \;\; \frac{1}{G_d}\frac{\rho^{x_k}(R\rho)^{z_k}}{x_k! z_k!} \tag{17}$$

where $0 \leq x_k \leq C$, and $0 \leq z_k \leq C - x_k$. G_d denotes the normalization constant given by:

$$G_d = \sum_{x_k=0}^{C} \sum_{z_k=0}^{C-x_k} \frac{\rho^{x_k}(R\rho)^{z_k}}{x_k! z_k!} \tag{18}$$

Along the same line as for the omnidirectional case, the probability that satisfies Equation 4, denoted by P_{sd}, may be written as:

$$\begin{aligned} P_{sd} &= \pi(W_{i-R} < C) \prod_{k=i-R}^{i-1} P(W_{k+1} < C | W_k < C) \\ &= \pi(W_{i-R} < C) P(W_{k+1} < C | W_k < C)^R \end{aligned} \tag{19}$$

The blocking probability for the line network employing directional antennas is given by:

$$P_{bd} = 1 - P_{sd}. \tag{20}$$

V. PERFORMANCE EVALUATION

We evaluate the effectiveness of the developed analytical model by comparing the blocking probability of a call obtained using the analytical model with that of simulation. We consider a 50-node line network and evaluate the blocking performance by varing the call arrival rate and the transmission range of the nodes. We assume that the nodes have sufficient number of transceivers as necessary, hence there is no blocking due to transceiver unavailability.

For a given call request from node s, we check if conditions specified in Equations 2 for $i \in \{s-2R, ..., s\}$ and 4 for $i \in \{s-R, ..., s\}$ for the networks employing omnidirectional and directional antennas, respectively. If it is satisfied, the call is accepted otherwise it is blocked. To remove the end effects of the line network, we consider only the blocking of calls originating from nodes 16 to 34 in the network.

Figure 1 shows the blocking probability versus call carrival rate for the considered line network when C=1 and transmission range varying from 1 to 3 units. The analytical model approximates the blocking probability very close to the actual simulated values for varying transmission ranges. The model provides efficient approximation even if number of channels increases. Figure 2 shows the blocking probability for 10 channels and transmission ranges 1, 2, and 3.

As expected, the employment of directional antenna reduces the blocking probability. The gain in blocking probability due to directional antenna increases with the number of channels available in the network. It can be seen in Figure 2 that for average load of 1, the difference in blocking probability is more than an order of magnitude.

VI. CONCLUSION

In this paper, we analyze rearrangement channel assignment policy in a multi-channel wireless line network. We considered networks employing omnidirectional antennas as well as directional antennas. We showed through simulations that the analytical model approximates very well to the simulated blocking probability for single channel as well as multiple channels. We also showed that the blocking probability is reduced employing directional antenna. In addition, the blocking probability of a call decreases signficantly as the number of channels in the network increases.

In this paper, we assumed that the interference range is the same as transmission range. However, in practice, the former is larger than the latter. In addition, the analysis presented in this paper is restricted to single-hop calls. In practice, calls may be routed over multiple hops. In such scenarios, the channel assignement for calls will have to be distinct over three and two successive hops for omindirectional and directional anntenna scenarios, respectively. As part of the future work, we plan to extend the developed analytical model to account for the above practical aspects.

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REFERENCES

[1] I. F. Akyildiz, X. Wang, and W. Wang, "Wireless mesh networks: a survey," in *Computer Networks*, March 2004, vol. 47, pp. 445–487.

[2] R. Bruno, M. Conti, and E. Gregori, "Mesh networks: commodity multihop ad hoc networks," in *IEEE Communications Magazine*, March 2005, vol. 43, pp. 123–131.

[3] N. Mastronarde, Y. Andreopoulos, M. V. D. Schaar, D. Krishnaswamy, and J. Vicente, "Cross-layer video streaming over 802.11e-enabled wireless mesh networks," in *ICASSP*, 2006, vol. 5, pp. V–V.

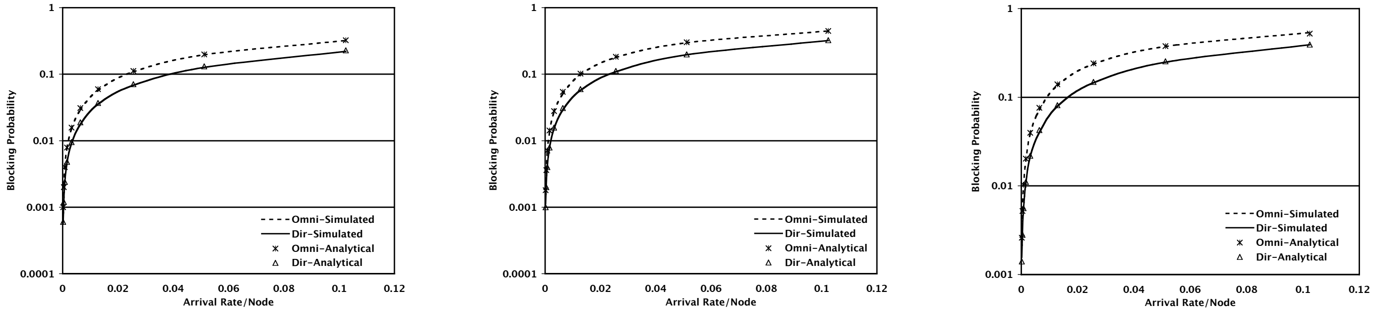
[4] G. A. Kennedy and P. J. Foster, "High resilience networks and microwave propagation in underground mines," in *The 9th European Conference on Wireless Technology*, September 2006, pp. 193–196.

[5] A. P. Subramanian, H. Gupta, and S. R. Das, "Minimum interference channel assignment in multi-radio wireless mesh networks," in *SECON*, June 2007, pp. 481–490.

[6] K. N. Ramachandran, E. M. Belding, K. C. Almeroth, and M. M. Buddhikot, "Interference-aware channel assignment in multi-radio wireless mesh networks," in *INFOCOM*, April 2006, pp. 1–12.

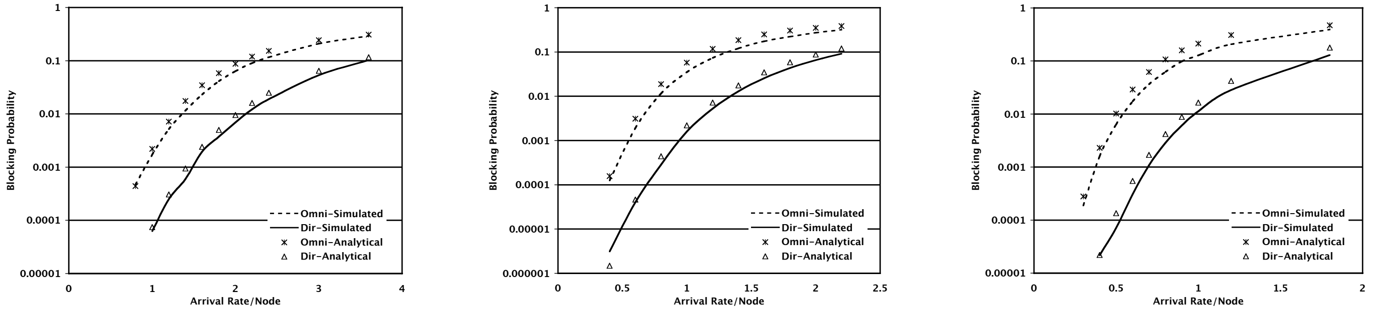
[7] S. O. Krumke, M. V. Marathe, and S. S. Ravi, "Models and approximation algorithms for channel assignment in radio networks," in *Wireless Networks*, November 2001, vol. 7, pp. 575–584.

[8] A. A. Bertossi, M. C. Pinotti, and R. Rizzi, "Channel assignment on strongly-simplicial graphs," in *International Parallel and Distributed Processing Symposium*, 2003, p. 8.



(a) C = 1, R = 1 (b) C = 1, R = 2 (c) C = 1, R = 3

Fig. 1. Blocking probability vs. Arrival rate per node for channel = 1 and transmission range = 1, 2, and 3.



(a) C = 10, R = 1 (b) C = 10, R = 2 (c) C = 10, R = 3

Fig. 2. Blocking probability vs. Arrival rate per node for channel = 10 and transmission range = 1, 2, and 3.

[9] L. Xu, Y. Xiang, and M. Shi, "A novel channel assignment algorithm based on topology simplification in multi-radio wirelesss mesh networks," in *25th IEEE International Performance, Computing, and Communications Conference*, April 2006, p. 8.

[10] C.-C. Hsu, P. Liu, D.-W. Wang, and J.-J. Wu, "Generalized edge coloring for channel assignment in wireless networks," in *International Conference on Parallel Processing*, 2006, pp. 82–92.

[11] C. L. Barrett, G. Istrate, V. S. A. Kumar, M. V. Marathe, S. Thite, and S. Thulasidasan, "Strong edge coloring for channel assignment in wireless radio networks," in *Fourth Annual IEEE International Conference on Pervasive Computing and Communications Workshops*, March 2006, p. 106.

[12] H.-J. Huang, X.-L. Cao, X.-H. Jia, and X.-L. Wang, "A bibd-based channal assignment algorithm for multi-radio wireless mesh networks," in *International Conference on Machine Learning and Cybernetics*, August 2006, pp. 4419–4424.

[13] M. Shin, S. Lee, and Y.-A. Kim, "Distributed channel assignment for multi-radio wireless networks," in *MASS*, October 2006, pp. 417–426.

[14] S. Yi, Y. Pei, and S. Kalyanaraman, "On the capacity improvement of ad hoc wireless networks using directional antennas," in *ACM international symposium on Mobile ad hoc networking and computing*, 2003, pp. 108–116.

[15] J. Zhang and S. C. Liew, "Capacity improvement of wireless ad hoc networks with directional antennae," in *IEEE Vehicular Technology Conference*, May 2006, vol. 2, pp. 911–915.

[16] A. Spyropoulos and C. S. Raghavendra, "Capacity bounds for ad-hoc networks using directional antennas," in *IEEE International Conference on Communications*, May 2003, vol. 1, pp. 348–352.

[17] S. Ramanathan, "A unified framework and algorithm for channel assignment in wireless networks," in *Wireless Networks*, 1999, vol. 5, pp. 81–94.

[18] R. Ramanathan, J. Redi, C. Santivanez, D. Wiggins, and S. Polit, "Ad hoc networking with directional antennas: A complete system solution," *JSAC*, vol. 23, no. 3, pp. 496–506, March 2005.

[19] D. Everitt and D. Manfield, "Performance analysis of cellular mobile communication systems with dynamic channel assignment," in *JSAC*, October 1989, vol. 7, pp. 1172–1180.

[20] M. Zafer and E. Modiano, "Blocking probability and channel assignment in wireless networks," in *Transactions on Wireless Communications*, April 2006, vol. 5, pp. 869–879.