A Systematic Method to Generate Gray Code Sequences to Count Upto Arbitrary Even Numbers

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I. Background

The following problem was posted on Programmable Logic Design Line website on May 23, 2007 [PLD-1].

II. Problem Statement

Design a gray code counter to count up to a given number of words, N.

III. Solution

As the gray-code counter relies on one bit change at a time and the counter must be cyclic, gray code counters are possible only if N is an even number. Designing the gray code counter to count up to any power of 2 is straight-forward. We will assume that gray code counters are available to count up to any power of 2 to design our solution in a systematic manner. If not, it may also be generated from the systematic approach developed in this article.

Let M be the highest power of 2 that is lower than or equal to N. Let $X = \log M$.

1) Write out the X-bit gray code counter. If M = N, then we have our solution already. To represent more than M words, we need $X + 1$ bits. Therefore, if $N > M$, append a zero to the X-bit gray code values. The addition of a 0 to make the counter to $X + 1$ bits does not affect the gray code property. Call this sequence of values as the "M-word Gray Code Counter With an Appended 0" (M-GCC-A0). We need an additional (N - M) values added to the M-GCC-A0.
   Similarly, construct the M-word Gray Code Counter with an Appended 1" (M-GCC-A1).
2) Pair every consecutive two gray code values in M-GCC-A0 to obtain $\frac{M}{2}$ disjoint pairs. Similarly, obtain the corresponding pairs in M-GCC-A1. Let the two values in a pair belonging to M-GCC-A0 be denoted by (v_0, v'_0). Let the two values in the corresponding pair belonging to M-GCC-A1 be denoted by (v_1, v'_1).
   Observe that the pairs (v_0, v'_0), (v_1, v'_1), (v_0, v_1), and (v'_0, v'_1) all differ in only one bit position.
3) Assume that M-GCC-A0 will be used as the primary set of values for the GCC. Therefore, if only M values are required, the values in M-GCC-A0 will only be employed. (Of course, the 0 at the end may be removed in this case.). If we need to count up to more values than M, then we may include the additional values from M-GCC-A1.
   Observe that there are two distinct gray code sequences in each pair - (1) employing M-GCC-A0 values only; or (2) employing both M-GCC-A0 and M-GCC-A1 values. In the former case, the sequence is $v_0 \rightarrow v'_0$. In the latter case, the sequence is $v_0 \rightarrow v_1 \rightarrow v'_1 \rightarrow v'_0$. As we have $\frac{M}{2}$ such pairs, each pair can have a sequence length of either 1 or 3. If $N > M$, we need to add an additional $(N - M)$ values to the M-GCC-A0 sequence which is achieved by employing the longer sequence for any of the $\frac{(N-M)}{2}$ pairs among the $\frac{M}{2}$ pairs.

A. Example

We construct the gray code counters for $N = \{8, 10, 12, 14, 16\}$ from 3-bit gray code counters. Note that for $N = 16$, we obtain a 4-bit gray code counter from a 3-bit gray code counter. Therefore, the 3-bit gray code counter may be derived from 2-bit gray code counter in turn recursively.

Figure 1(a) shows the 8-GCC-A0 and 8-GCC-A1 values. Figures 1(b) through (f) show the one (possible) sequence of gray code counter to count upto 8, 10, 12, 14, and 16 words, respectively. [Note: In case of counting up to 8 words, the last zero can be omitted.] For example, the gray code sequence for counting up to 10 words shown in Figure 1(c) uses both 8-GCC-A0 and 8-GCC-A1 values for Pair 1 among the four possible scenarios. In addition, more alternatives may also be found if default 8-word gray code counter employs 8-GCC-A1 instead of 8-GCC-A2.

IV. Conclusion

In this article, a systematic method to construct gray code (cyclic) sequences for counting up to any given even number is developed. Interestingly, as gray-code sequences may be interpreted as walks along the edges of a hypercube, the sequence of gray codes counting up to N words represent cycles on hypercubes of with length N.

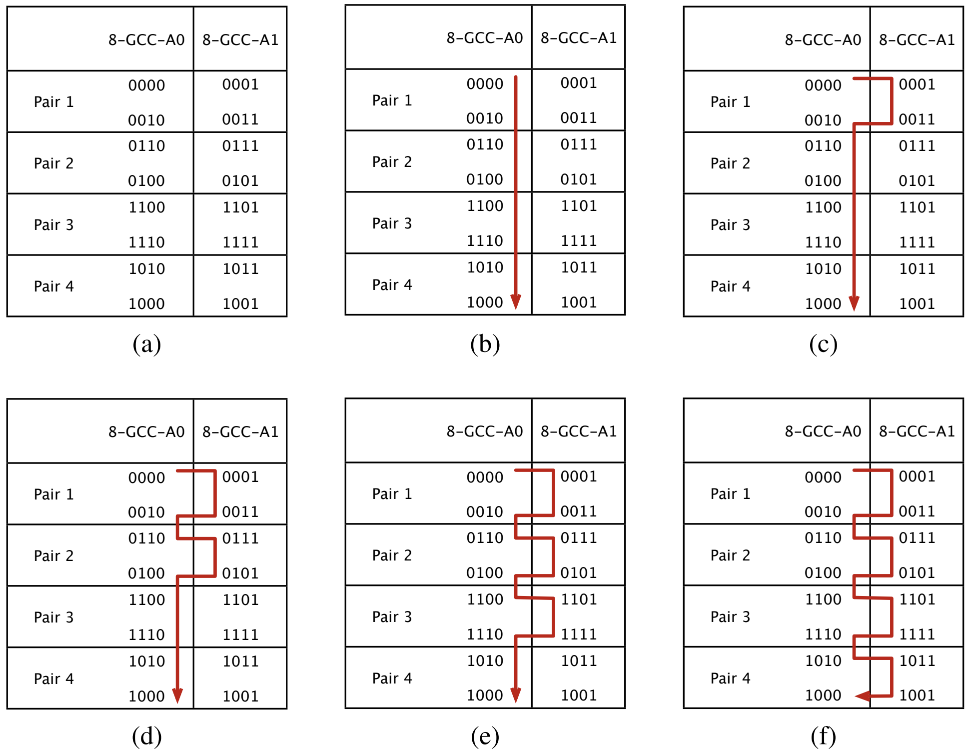

Fig. 1. (a) 8-GCC-A0 and 8-GCC-A1 values; (b) A sequence for 8-word gray code counter; (c) A sequence for 10-word gray code counter; (d) A sequence for 12-word gray-code counter; (e) A sequence for 14-word gray code counter; and (f) A sequence for 16-word gray code counter.

REFERENCES

[PLD-1] **An interesting gray code FIFO counter question:** http://pldesignline.com/199701541